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Effective Hamiltonian of Electroweak Penguin for Hadronic b Quark Decays

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Abstract

In this research we work with the effective Hamiltonian and the quark model. We investigate the decay rates of matter-antimatter of $\boldsymbol{b}$ quark. We describe the effective Hamiltonian theory and apply this theory to the calculation of current-current ($Q_{1,2}$), QCD penguin ($Q_{3,\dots,6}$), magnetic dipole (Q_8) and electroweak penguin ($Q'_{7,\dots,10}$) decay rates. The gluonic penguin structure of hadronic $\boldsymbol{b}$ decays $b - q_k g - q_k q_i q_{\bar{j}}$ is studied through the Wilson coefficients of the effective Hamiltonian. We calculate the branching ratios of the tree-level, effective Hamiltonian, effective Hamiltonian including electroweak Penguin, effective Hamiltonian including Magnetic Dipole and the effective Hamiltonian including electroweak Penguin and Magnetic Dipole $\boldsymbol{b}$ quark decays $b - q_i q_k q_{\bar{j}}$, $q_i \in \{u, c\}$, $q_k \in \{d, s\}$, $q_{\bar{j}} \in \{\bar{u}, \bar{c}\}$. We show that, the electroweak Penguin and Magnetic Dipole contributions in b quark decays are small and Current-Current operators are dominant.

Keywords: Effective Hamiltonian; b quark; Gluonic penguin; Electroweak penguin; Magnetic dipole

Introduction

In the Standard Model, flavor-changing neutral currents are forbidden, for example, there is no direct coupling between the b quark and the s or d quarks. Effective flavor-changing neutral currents are induced by one-loop, or "penguin" diagrams, where a quark emits and reabsorbs a W thus changing flavors twice, as in the $b \to t \to s$ transition. Penguin decays have become increasingly appreciated in recent years [1-3]. These loop diagrams with their interesting combination of CKM matrix elements give insight into the Standard Models [4]. In addition, they are quite sensitive to new physics. The weak couplings of the quarks are given by the CKM matrix. For the Standard Model with three generations, the CKM matrix can be described completely by three Euler-type angles, and a complex phase.

Various types of the penguin processes are [5]: electromagnetic, electroweak, and gluonic. In electromagnetic penguin decays such as $b \to s\gamma$, a charged particle emits an external real photon (see Fig.

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1). The hard photon emitted in these decays is an excellent experimental signature. The inclusive rate is dominated by short distance (perturbative) interactions and can be reliably predicted. The QCD corrections enhance the rate and have been calculated precisely. The electromagnetic penguin decay $b \to d\gamma$ is further suppressed by $|V_{td}|^2/|V_{ts}|^2$ and gives an alternative to $B^0-\bar{B}^0$ mixing for extraction $|V_{td}|$ [6,7]. Experimentally, inclusive $b \to d\gamma$ has large backgrounds from the dominate $b \to s\gamma$ decays which must be rejected using good particle identification or kinematics separation. The decay $b \to s\ell^+\ell^-$ can proceed via an electroweak penguin diagram where an emitted virtual photon or Z^0 produces a pair of leptons. This decay can also proceed via a box diagram (see Fig. 2) [8]. The Standard Model prediction for the $b \to s\ell^+\ell^-$ decay rate is two orders of magnitude smaller than the $b \to s\gamma$ rate [9,10]. The rate for $b \to s\nu\bar{\nu}$ is enhanced relative to $b \to s\ell^+\ell^-$ primarily due to summing the three neutrino flavors. These decays are expected to be dominated by the weak penguin, since neutrinos do not couple to photons. The predicted rate is only a factor of 10 lower than for $b \to s\gamma$ [2]. Unfortunately, the neutrinos escape detection, making this mode difficult to observe.

Another category of penguin is so-called vertical or annihilation penguin where the penguin loop connects the two quarks in the B meson. These rates are expected to be highly suppressed in the Standard Model since they involve a $b \to d$ transition and are suppressed by $(f_B/m_B)^2 \approx 2\times10^{-3}$ [11], where f_B is the B-meson decay constant which parameterizes the probability that the two quarks in the B meson will "find each other", and m_B is the B meson mass. The $B \to \gamma\gamma$ decay is suppressed [12] relative to $b \to s\gamma$ by an additional α_{QCD} (see Fig. 3). The $B \to \ell^+\ell^-$ decays are helicity-suppressed [13,7]. Because these decays are so suppressed in the Standard Model, they provide a good opportunity to look for non SM effects.

An on- or off-shell gluon can also be emitted from the penguin loop. While the on-shell $b \to sg$ rate had been calculated to be O(0.1%) [14], the inclusive on-plus off-shell $b \to sg^*$ rate includes contribution from $b \to sq\bar{q}$ and $b \to sgg$ which increase the inclusive rate to 0.5-1% [2,15]. The $b \to dg^*$ penguin rate is smaller by $|V_{td}/V_{ts}|^2$ (see Fig. 4).

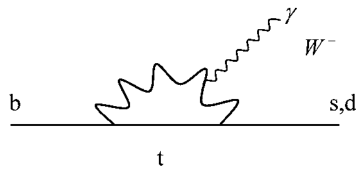

Figure 1. Feynman diagram for the electromagnetic penguin $b \to s\gamma$ and $b \to d\gamma$. The photon can be emitted from the W or from any of the quarks.

a)

b)

c)

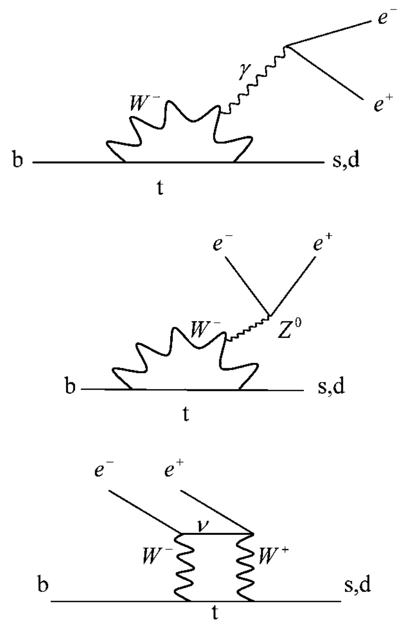

Figure 2. (a) photon penguin (b) Z^0 penguin and (c) box diagrams for the electroweak decay $b \to (s,d)\ell^+\ell^-$.The diagrams for $b \to (s,d)\nu\bar{\nu}$ are similar, except that (a) does not contribute.

(a)

(b)

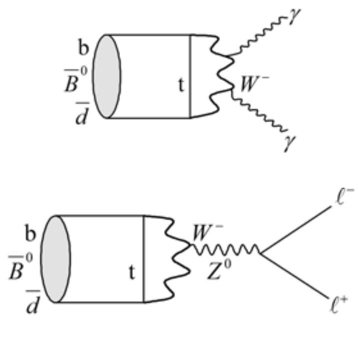

Figure 3. Vertical or annihilation penguins (a) $B \to \gamma\gamma$ and (b) $B \to \ell^+\ell^-$.

Unfortunately, there are several difficulties associated with gluonic penguins. There is no good signature for the inclusive $b \to sg^*$ decay, unlike the $b \to s\gamma$ case. The branching fraction of individual exclusive gluonic penguin channels is typically quite small and hadronization effects are difficult to calculate [5,16]. In addition, many gluonic penguin final states are accessible via other diagrams, so the gluonic penguin is difficult to assess. Thus the penguin processes such as $B^0 \to \varphi K^0$ that have contributions only from gluonic penguins are eagerly sought.

While the gluonic penguin gives rise only to hadronic final states, several other processes can contribute to the same final states. One important contribution is from the tree-level $b \to u$ decay. For example the $b \to us\bar{u}$ transition and the $b \to sg^*$ penguin transition both contribute to $B^0 \to K^+\pi^-$. However, the $b \to us\bar{u}$ transition is Cabibbo-suppressed, so the penguin process is expected to dominate [17-20]. On the other hand, in $B \to \pi^+\pi^-$ for example, the small $b \to dg^*$ contribution is expected to be dominant by the non-cabibbo-suppressed tree-level $b \to ud\bar{u}$ transition. In general, most decays to hadronic final state with φ mesons or non-zero net strangeness are expected to be dominate by gluonic penguin and hadronic final states with zero net strangeness are expected to be dominated by tree-level $b \to u$.

Electroweak penguin also contributes to hadronic final states. Every gluonic penguin can be converted to an electrowrak penguin by replacing the gluon with a Z^0 or γ (see Fig. 2). Electroweak penguin with internal Z^0 or γ emission are suppressed relative to the corresponding strong gluonic penguin. In the hairpin process the gluon, Z^0, or γ is emitted externally and subsequently forms a meson.

The vertical electroweak penguin diagram, with the lepton pair replaced by a de-quark pair, is highly suppressed and is important only for decays such as $B^0 \to \varphi\varphi$, where no other diagrams contribution [21,22]. In the annihilation diagram the b and $\bar{u}$ quarks in a B^- meson annihilate to form a virtual W^-. The annihilation diagram is suppressed by $|V_{ub}|$ and by f_B / m_B and is expected to be mostly negligible. In the exchange diagram, a $b \to u$ transition and a $\bar{d} \to \bar{u}$ transition occur simultaneously via the exchange of a W between the b and $\bar{d}$ quarks in a $\bar{B}^0$ meson. The exchange process is also suppressed by $|V_{ub}|$ and f_B / m_B , and is also expected to be negligible, except in decays such as $B^0 \to K^+K^-$ where no favored diagrams contribution [23-25].

Although $s \to u$ loop diagrams are important in K decays, those decays are typically dominated by large non-perturbative effects. A notable exception is $K^+ \to \pi^+\nu\bar{\nu}$. This decay is expected to be dominated by electroweak penguins and could eventually provide a measurement of $|V_{td}|$. Penguin processes are also possible in c and t decays, but these particles have the CKM-favored decays $c \to s$ and $t \to b$ accessible to them. Since the b quark has no kinematically-allowed CKM-favored decay, the relative importance of the penguin decay is greater. The mass of the top quark the main contributor to the loop, is large, and the coupling of the b quark, the t quark, $|V_{tb}|$, is very close to unity, both strengthening the effect of the penguin. The $b \to s(b \to d)$ penguin transition is sensitive to $|V_{ts}|/|V_{td}|$ which will be extraordinarily difficult to measure in top decay. Information from the penguin decay will complement information on $|V_{ts}|$ and $|V_{td}|$ from $B_s - \bar{B}_s$ and $B^0 - \bar{B}^0$ mixing [26]. Since the Standard Model loops involve the heaviest know particles (t ,W ,Z), rates for these processes are very sensitive to non-SM extension with heavy charged Higge or supersymmetric particle. Therefore, measurement of loop processes constitutes the most sensitive low energy probes for such extensions to the Standard Model.

Conservation of the gluonic current requires the $b \to q_k g$ vertex to have the structure [27,28]:

$$\Gamma^a_\mu(q^2) = (ig_s / 4\pi^2)\bar{u}_k(p_k)T^a V_\mu(q^2)u_b(p_b) . \quad (1)$$

where

$$V_\mu(q^2) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\gamma^\nu[F_1^L(q^2)P_L + F_1^R(q^2)P_R] + i\sigma_{\mu\nu}q^\nu[F_2^L(q^2)P_L + F_2^R(q^2)P_R] . \quad (2)$$

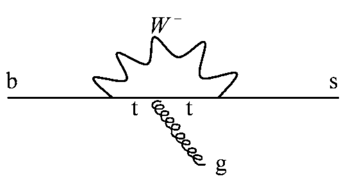

Figure 4. Feynman diagram for the gluonic penguin $b \to sg^*$ .The gluon can be emitted from any of the quark lines and can be on-shell or off-shell.

Here and F_2 are the electric (monopole) and magnetic (dipole) form factors, $q = q_g = p_b - p_k$ is the gluon four momentum, $P_{L(R)} \equiv (1 \mp \gamma_5)/2$ are the chirality projection operators and T^a $(a = 1,...,8)$ are the $SU(3)_c$ generators normalized to $Tr(T^a T^b) = \delta^{ab}/2$. The $\bar{b} \to \bar{q}_k g$ vertex is

$$\bar{\Gamma}^a_\mu(q^2) = -(ig_s/4\pi^2)\bar{v}_b(p_b)T^a\bar{V}_\mu(q^2)v_k(p_k). \quad (3)$$

Here $\bar{V}_\mu$ has the form (2) with the form factors $F^{L,R}_{1,2}(q^2)$ replaced by $\bar{F}^{L,R}_{1,2}(q^2)$. To lowest order in α_s the penguin amplitude for the decay process $b \to q_k g \to q_k q' \bar{q}'$ $(q_k q_i \bar{q}_j)_{i=j}$ is

$$M^{Peng} = -i(\alpha_s/\pi)[\bar{u}_k(p_k)T^a\Lambda_\mu u_b(p_b)] \\ [\bar{u}_{q'}(p_{q'})\gamma^\mu T^a v_{\bar{q}'}(p_{\bar{q}'})]. \quad (4)$$

where $\alpha_s = g_s^2/4\pi$ and

$$\Lambda_\mu \equiv \gamma_\mu[F_1^L(q^2)P_L + F_1^R(q^2)P_R] \\ +(i\sigma_{\mu\nu}q^\nu/q^2)[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \quad (5)$$

Similarly, for $\bar{b} \to \bar{q}_k q' \bar{q}'$, the amplitude is

$$\bar{M}^{Peng} = i(\alpha_s/\pi)[\bar{v}_k(p_k)T^a\bar{\Lambda}_\mu v_b(p_b)] \\ [\bar{u}_{q'}(p_{q'})\gamma_\mu T^a v_{\bar{q}'}(p_{\bar{q}'})]. \quad (6)$$

Where $\bar{\Lambda}_\mu$ is obtained from (5) by the replacement of all the $F(q^2)$ form factors by $\bar{F}(q^2)$ form factors. The top quark dominates in the sum for F_2, hence at value of q^2 (a good approximation), we have $F_2^L(q^2) \approx F_2^L(0)$ and $F_2^R(q^2) \approx F_2^R(0)$ [29], so

$$F_1^L(q^2) = (G_F/\sqrt{2})\sum_{i=u,c,t} V_{ik}^* V_{ib} f_1(x_i, q^2),$$

$$F_1^R(0) = 0 \quad (7)$$

$$F_2^L(0)/m_q = F_2^R(0)/m_b = (G_F/\sqrt{2})\sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i). \quad (8)$$

Where $x_i \equiv m_i^2/M_W^2$ $(i = u,c,t)$ and

$$f_2(x) = -(x/4(1-x)^4)[2+3x-6x^2+x^3+6x\ln x]. \quad (9)$$

$$f_1(x) = (1/12(1-x)^4)[18x - 29x^2 + 10x^3 \\ +x^4 - (8 - 32x + 18x^2)\ln x]. \quad (10)$$

$$f_1(x_i, q^2) = (10/9) - (2/3)\ln x_i + (2/3z_i) \\ -(2(2z_i+1)/3z_i)g(z_i) \quad (11)$$

Here $z_i \equiv q^2/4m_i^2$ and

$$g(z) = \begin{cases} \sqrt{\frac{1-z}{z}}\arctan(\sqrt{\frac{z}{1-z}}), & z < 1 \\ \frac{1}{2}\sqrt{\frac{z-1}{z}}[\ln(\frac{\sqrt{z}+\sqrt{z-1}}{\sqrt{z}-\sqrt{z-1}}) - i\pi], & z > 1 \end{cases} \quad (12)$$

For the u quark, z_i is large and we use the asymptotic form of (11),

$$f_1(x_u, q^2) = (10/9) - (2/3)[\ln(q^2/M_W^2) - i\pi]. \quad (13)$$

We find $F_1^L >> F_1^R$ and $F_2^R >> F_2^L$. For the $b \to dq'\bar{q}'$ amplitude we find that F_1^L is dominant. Processes like $b \to ds\bar{s}$ and $\bar{b} \to \bar{d}s\bar{s}$ are expected to be penguin dominated [30] and F_1^L dominates all the other form factors. In the $b \to sq'\bar{q}'$ transition, we again find that $F_1^L >> F_1^R$, $F_2^R >> F_2^L$ and the F_1^L amplitude to be dominant.

Now, a very important issue is the generation of QCD corrections to penguin operators. Consider for example, the local operator $(\bar{u}_\alpha b_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A}$, which is directly induced by W-boson exchange. In this case, additional QCD correction diagrams, with a gluon contribute and as a consequence four operators are involved in the mixing under renormalization instead of two. These are [31,32]:

$$Q_3 = (\bar{d}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V-A},$$

$$Q_4 = (\bar{d}_\beta b_\alpha)_{V-A}\sum_q(\bar{q}_\alpha q_\beta)_{V-A},$$

$$Q_5 = (\bar{d}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V+A},$$

$$Q_6 = (\bar{d}_\beta b_\alpha)_{V-A}\sum_q(\bar{q}_\alpha q_\beta)_{V+A}, \quad (14)$$

α and β are colour indices. The sum over q runs over all quark flavors that exist in the effective theory in question. Since the gluon coupling is of course flavor conserving, it is clear that penguins cannot be generated from the operator current due to the gluon coupling in the lower part. For convenience this vector structure is decomposed into a (V-A) and a (V+A) part according to chiral representation,

$$(\bar{q}_{i\alpha}b_{\alpha})_{V\mp A}(\bar{q}_{k\beta}q_{j\beta})_{V\mp A}$$
$$=(\bar{q}_{i\alpha}\gamma^{\mu}((1\mp\gamma_5)/2)b_{\alpha})(\bar{q}_{k\beta}\gamma_{\mu}((1\mp\gamma_5)/2)q_{j\beta}). \tag{15}$$

and in the terms of two component spinors are given by,

$$\bar{q}_{i\alpha}\gamma^{\mu}((1-\gamma_5)/2)b_{\alpha}=q^{\dagger}_{i\alpha L}\tilde{\sigma}^{\mu}b_{\alpha L},$$
$$\bar{q}_{k\beta}\gamma_{\mu}((1-\gamma_5)/2)q_{j\beta}=q^{\dagger}_{k\beta L}\tilde{\sigma}_{\mu}q_{j\beta L},$$
$$\bar{q}_{i\alpha}\gamma^{\mu}((1+\gamma_5)/2)b_{\alpha}=q^{\dagger}_{i\alpha R}\sigma^{\mu}b_{\alpha R},$$
$$\bar{q}_{k\beta}\gamma_{\mu}((1+\gamma_5)/2)q_{j\beta}=q^{\dagger}_{k\beta R}\sigma_{\mu}q_{j\beta R}, \tag{16}$$

For each of these, two different colour forms arise due to the colour structure of the exchanged gluon. The amplitude (4) can be written [33],

$$M^{Peng}=-i(G_F/\sqrt{2})\{(\alpha_s(M_W/8\pi)$$
$$[\sum_{i=u,c}V_{ib}V^{*}_{iq}f_1(x_i,q^2)+V_{tb}V^{*}_{tq}f_1(x_t)]\,Q_P \tag{17}$$
$$+(1/2)\sum_{i=u,c,t}V_{ib}V^{*}_{iq}f_2(x_i)\,Q_8\}$$

Q_8 is the chromomagnetic dipole operator:

$$Q_8=4\alpha_s^2m_b[\bar{q}_{i\alpha}\sigma^{\mu\nu}(1+\gamma_5)T^{a}_{\alpha\beta}b_{\beta}]$$
$$(q_{\nu}/q^2)[\bar{q}_{\bar{j}}\gamma_{\mu}T^{a}q_k]. \tag{18}$$

Here

$$Q_P=Q_4+Q_6-(1/3)(Q_3+Q_5). \tag{19}$$

As a weak decay in the presence of the strong interaction B meson decays require special techniques [34]. The main tool to calculate such B meson decays is the effective Hamiltonian theory [35,36]. It is a two step program, starting with an operator product expansion (OPE) and performing a renormalization group equation (RGE) analysis afterwards [36-38]. The necessary machinery has been developed over the last years.

The derivation starts as follows: If the kinematics of the decay are of the kind that the masses of the internal particle M_i are much larger than the external momenta P, $M_i^2>>p^2$, then the heavy particle can be integrated out. This concept takes concrete form with the functional integral formalism. It means that the heavy particles are removed as dynamical degrees of freedom from the theory hence their fields do not appear in the (effective) Lagrangian anymore. Their residual effect lies in the generated effective vertices [39]. In this way an effective low energy theory can be constructed from a full theory like the Standard Model [40]. A well known example is the four-Fermi interaction, where the W-boson propagator is made local for $M_W^2>>q^2$ (q denotes the momentum transfer through the W):

$$-i(g_{\mu\nu})/(q^2-M_W^2)$$
$$\rightarrow ig_{\mu\nu}[(1/M_W^2)+(q^2/M_W^4)+...], \tag{20}$$

Where the ellipses denote terms of higher order in $1/M_W$.

Apart from the t quark the basic framework for weak decays quarks is the effective field theory relevant for scales $M_W,M_Z,M_t>>\mu$ [35,41]. This framework, as we have seen above, brings in local operators, which govern "effectively" the transition in question. From the point of view of the decaying quark, it represents the generalization of the Fermi theory as formulated by Sudershan and Marshak and Feynman and Gell-Mann forty years ago.

It is well known that the decay amplitude is the product of two different parts, whose phases are made of a weak (Cabbibo-Kobayashi-Maskawa) and a strong (final state interaction) contribution. The weak contributions to the phases change sign when going to the CP-conjugate process, while the strong ones do not. Indeed the simplest effective Hamiltonian without QCD effects ($b\rightarrow ud\bar{u}$) is

$$H^{0}_{eff}=2\sqrt{2}G_F\,V_{ub}V^{*}_{ud}\,Q_1, \tag{21}$$

where G_F is the Fermi constant, V_{ij} are the relevant CKM factors and

$$Q_1=(\bar{u}_{\alpha}b_{\alpha})_{V-A}(\bar{d}_{\beta}u_{\beta})_{V-A}, \tag{22}$$

is a $(V-A)$, $(V-A)$ is current-current local operator. This simple tree amplitude introduces a new operator Q_2 and is modified by the QCD effect to

$$H_{eff}=2\sqrt{2}G_F\,V_{ub}V^{*}_{ud}(C_1Q_1+C_2Q_2), \tag{23}$$

Here

$$Q_2=(\bar{u}_{\beta}b_{\alpha})_{V-A}(\bar{d}_{\alpha}u_{\beta})_{V-A}. \tag{24}$$

where C_1 and C_2 are the Wilson coefficients. The situation in the Standard Model is, however, more complicated because of the presence of additional interactions in particular penguins which effectively generate new operators. These are in particular the gluon, photon and Z^0-boson exchanges and penguin b quark contributions.

Consequently the relevant effective Hamiltonian for

B-meson decays involves generally several operators Q_i with various colour and Dirac structures which are different from Q_1. The operators can be grouped into three categories [42]: $i = 1,2-$ current-current operators, $i = 3,...,6,8-$ gluonic penguin operators and $i = 7',...,10'-$ Electroweak Penguin operators. Moreover each operator is multiplied by a calculable Wilson coefficient $C_i(\mu)$:

$$H_{eff} = 2\sqrt{2}G_F[\sum_{i=1}^{6,8} d_i(\mu)Q_i(\mu) + \sum_{i=7}^{10} d_i'(\mu)Q_i'(\mu)], \quad (25)$$

Where the scale μ is discussed below, $d_i(\mu) = V_{CKM}C_i(\mu)$ and V_{CKM} denotes the relevant CKM factors that are:

$$d_{1,2} = V_{ib}V_{jk}^*C_{1,2},$$

$$d_{3,...,6} = -V_{tb}V_{tk}^*C_{3,...,6},$$

$$d_8 = -(\alpha_s/4\pi)V_{tb}V_{tk}^*C_8,$$

$$d'_{7,...,10} = -(3/2)e_q V_{tb}V_{tk}^*C_{7,...,10}. \quad (26)$$

For tree-level the $d_{1,2}$ coefficients are:

$$d_1 = V_{ib}V_{kj}^*, \qquad d_2 = 0. \quad (27)$$

and for the effective Penguin Model the $d_{3,...,6,8}$ coefficients are:

$$d_3 = d_5 = (1/6)(\alpha_s/4\pi)\sum_{i=u,c,t} V_{iq}^*V_{ib}f_1(x_i),$$

$$d_4 = d_6 = -3d_3,$$

$$d_8 = -(m_b/2)(\alpha_s/4\pi)\sum_{i=u,c,t} V_{iq}^*V_{ib}f_2(x_i). \quad (28)$$

At this stage it should be mentioned that the usual Feynman diagram containing full W propagators, Z^0 propagators and top quark propagators represent really the happening at scales $O(M_W)$ whereas the true picture of a decaying hadron is more correctly described by the local operators in question. Thus, whereas at scale $O(M_W)$, we have to deal with the full six-quark theory containing the photon, weak gauge bosons and gluon, at scale $O(1GeV)$ the relevant effective theory contains only three light quarks u,d, and s, gluons and the photon. At intermediate energy scales $\mu = O(m_b)$ and $\mu = O(m_c)$ relevant for beauty and charm decays, effective five-quark and effective four-quark theories have to be considered, respectively [43].

The usual procedure then is to start at a high energy scale $O(M_W)$ and consecutively integrate out the heavy degrees of freedom (heavy with respect to the relevant scale μ) from explicitly appearing in the theory. The word explicitly is very essential here. The heavy field did not disappear. Their effects are merely hidden in the effective gauge coupling constants, running masses and most importantly the coefficients describing the effective strength of the operators at a scale μ, the Wilson coefficient functions $C_i(\mu)$ [31,35,36,44]. It is straightforward to apply H_{eff} to B- and D-meson decays as well by changing the quark flavors appropriately. μ is some low-energy scale of $O(1GeV)$, $O(m_c)$ and $O(m_b)$ for K, D, and B meson decays, respectively. The argument μ of the operators $Q_i(\mu)$ means that their matrix elements are to be normalized at scale μ.

In this research we obtained the decay rates of the $b \to q$ particle and $\bar{b} \to \bar{q}$ antiparticle for the various transitions at the:

Tree-Level.
Penguin.
Effective Hamiltonian.
Effective Hamiltonian including Electroweak Penguin.
Effective Hamiltonian including Magnetic Dipole.
Effective Hamiltonian including Electroweak Penguin and Magnetic Dipole.

Materials and Methods

A) Magnetic Dipole Amplitude of $b \to q_i q_k \bar{q}_j$

A charge particle in orbital motion generates a magnetic dipole moment of a magnitude proportional to its orbital angular momentum. Further more, a particle with intrinsic angular momentum or spin has an intrinsic magnetic moment. The magnetic dipole term in the penguin amplitude, according to (5), is

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu}q^\nu/q^2)[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \quad (29)$$

Also, according to (8) magnetic (dipole) form factor at $q^2 = 0$ ($q^2/M_W^2 << 1$) is

$$\frac{F_2^L(0)}{m_k} = \frac{F_2^R(0)}{m_b} = \frac{g_2^2}{8M_W^2}\sum_i V_{ik}^*V_{ib}f_2(x_i). \quad (30)$$

The top quark is dominant for $F_2^R(0)$, so we can write

$$F_2^R(0) = m_b(G_F/\sqrt{2})(V_{tk}^*V_{tb})f_2(x_t). \quad (31)$$

Here $f_2(x_t)$ defined by (9) and $x_t = m_t^2 / M_W^2$, also we saw that $F_2^L(0) << F_2^R(0)$, because $m_k << m_b$ so the magnetic dipole term becomes to

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu} q^\nu / q^2) F_2^R(0) P_R \ . \tag{32}$$

Putting in the penguin amplitude, according to (4),

$$M^{dip} = \frac{g_s^2}{4\pi^2} [\bar{u}_k(p_k) T^a (i\sigma_{\mu\nu} q^\nu / q^2) F_2^R(0) P_R u_b(p_b)][\bar{u}_i(p_i)\gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})]. \tag{33}$$

The magnetic dipole of penguin amplitude is given by (see App.A),

$$M^{dip} = A_8 d_8 [\langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle + \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle]. \tag{34}$$

Here

$$d_8 = -(2\sqrt{2} G_F)(m_b / 2)(\alpha_s / 4\pi) \sum_i V_{ik}^* V_{ib} f_2(x_i) \ .$$

$$A_8 = (1/\sqrt{2})(4/3)(m_b / q^2) \ . \tag{35}$$

Now we must calculate each terms of above equation for b spins project -1/2 and 1/2 then squaring these terms and adding all of them and at least averaging. The penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by (see App.A),

$$\begin{aligned} M^{dip}_{(+1/2)} = A_8 d_8 \{ & [-\sin((\theta_b + \theta_i - \theta_k - \theta_j)/2) \\ & + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \\ & + [-\sin((\theta_b - \theta_i - \theta_k + \theta_j)/2) \\ & + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \} \end{aligned} \tag{36}$$

$$\begin{aligned} M^{dip}_{(-1/2)} = A_8 d_8 \{ & [-\cos((\theta_b - \theta_i - \theta_k + \theta_j)/2) \\ & - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \\ & + [-\cos((\theta_b + \theta_i - \theta_k - \theta_j)/2) \\ & - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \} \end{aligned} \tag{37}$$

B) Effective Hamiltonian Decay Rates of $b \to q_i q_k \bar{q}_j$

The effective $\Delta B = 1$ Hamiltonian at scale $\mu = \mathrm{O}(m_b)$ for tree plus penguin and including the electroweak penguin and the magnetic dipole term is [31,32,35],

$$\begin{aligned} H_{eff}^{\Delta B=1} = 2\sqrt{2} G_F \{ & [d_{1c}(\mu) Q_1^c(\mu) + d_{2c}(\mu) Q_2^c(\mu)] \\ & + [d_{1u}(\mu) Q_1^u(\mu) + d_{2u}(\mu) Q_2^u(\mu)] \\ & - [\sum_{i=3}^{6,8} d_i(\mu) Q_i(\mu) + \sum_{i=7}^{10} d_i'(\mu) Q_i'(\mu)] \} \ . \end{aligned} \tag{38}$$

Here $d_1,\ldots,_6, d_7',\ldots,_{10}$ are defined by (26), $d_{1,2c,u} = d_{1,2}(i = j = c,u)$ and index k refer to d or s. The decay rate is given by (see App.B)

$$d^2\Gamma_{Q_1,\ldots,Q_6} / dxdy = \Gamma_{0b} I_{ps}^{EH} \ . \tag{39}$$

here

$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3 \ . \tag{40}$$

where

$$\begin{aligned} I_{ps}^1 &= 6xy . f_{ab} .(1 - h_{abc}) \ , \\ I_{ps}^2 &= 6xy . f_{bc} .(1 + h_{bca}) \ , \\ I_{ps}^3 &= 6xy . f_{ac} . h_{xa} . h_{yc} \ . \end{aligned} \tag{41}$$

and

$$\begin{aligned} \alpha_1 &= |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2 \ , \\ \alpha_2 &= |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2) \ , \\ \alpha_3 &= \mathrm{Re}\{(3d_1 + d_2 + d_3 + 3d_4) d_6^* \\ & \quad + (d_1 + 3d_2 + 3d_3 + d_4) d_5^* \} \ . \end{aligned} \tag{42}$$

Here Γ_{0b}, f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{xa} and h_{yc} defined in Appendix B.

Tree-Level: Before anything else, we can obtain decay rates of tree-level, without QCD corrections, from (39), if we choose $d_1 = V_{ib} V_{jk}^*$ and $d_2 = d_3 = \ldots = d_6 = 0$, so $\alpha_1 = 3d_1^2, \alpha_2 = 0$ and $\alpha_3 = 0$, Q_1 is the conventional four-Fermi interaction operator thus (39) reduce to $\Gamma_{Q_1,\ldots,Q_6} = \Gamma_{0b} I_{ps}^{EH}$ ($I_{ps}^{EH} = \alpha_1 I_{ps}^1$), so

$$\Gamma_{tree} = 3\Gamma_{0b} |V_{ib} V_{jk}^*|^2 I_{ps}^1 \ . \tag{43}$$

Pure Penguin: Also we can obtain the decay rates of pure penguin, if we choose $d_1 = d_2 = 0$, so α_1 and α_3 reduce to,

$$\begin{aligned} \alpha_1 &= |d_3 + d_4|^2 + 2|d_3|^2 + 2|d_4|^2 \ , \\ \alpha_3 &= \mathrm{Re}\{(d_3 + 3d_4) d_6^* + (3d_3 + d_4) d_5^* \} \ . \end{aligned} \tag{44}$$

α_2 and the decay rate keep the form (42).

C) Effective Hamiltonian of Electroweak Penguin Decay Rates

The generalization of the $\Delta B=1$ Hamiltonian in pure QCD to incorporate electroweak penguin operators is the sum of the $Q_1,...,Q_6,Q_7',...,Q_{10}'$ (38). The $\Delta B=1$ Wilson coefficients for $Q_1^{u,c},Q_2^{u,c},Q_3,...,Q_6,Q_7',...,Q_{10}'$ in the mixed case of QCD and QED. Therefore the discussion of $C_1,...,C_6$ is also valid for the present case. We saw that, all of the terms Q_1,Q_2,Q_3,Q_4 have a form Left-Left handed. Terms Q_1,Q_2 have a form $\langle i\left|_L \tilde{\sigma}^\mu \right|b\rangle_L \langle k\left|_L \tilde{\sigma}_\mu \right| j\rangle_L$, and terms Q_3,Q_4 have a form $\langle k\left|_L \tilde{\sigma}^\mu \right|b\rangle_L \langle i\left|_L \tilde{\sigma}_\mu \right| j\rangle_L$. Also terms Q_5,Q_6 have a form Left-Right handed, $\langle k\left|_L \tilde{\sigma}^\mu \right|b\rangle_L \langle i\left|_R \sigma_\mu \right| j\rangle_R$. We consider that terms Q_7',Q_8' have a form Left-Right handed and terms Q_9',Q_{10}' have a form Left-Left handed. Therefore terms Q_7',Q_8' have a form $\langle k\left|_L \tilde{\sigma}^\mu \right|b\rangle_L \langle i\left|_L \tilde{\sigma}_\mu \right| j\rangle_L$, and terms Q_9',Q_{10}' have a form $\langle k\left|_L \tilde{\sigma}^\mu \right|b\rangle_L \langle i\left|_R \sigma_\mu \right| j\rangle_R$. Thus the partial decay rate including electroweak penguin is the same as (39) with different constants α_1,α_2 and α_3,

$$\alpha_1 = \left|d_1+d_2+d_3+d_4+d_9'+d_{10}'\right|^2 + 2\left|d_1+d_4+d_{10}'\right|^2 + 2\left|d_2+d_3+d_9'\right|^2 ,$$

$$\alpha_2 = \left|d_5+d_6+d_7'+d_8'\right|^2 + 2\left|d_5+d_8'\right|^2 + 2\left|d_6+d_7'\right|^2) .$$

$$\alpha_3 = \mathrm{Re}\{(3d_1+d_2+d_3+3d_4+d_9'+3d_{10}')(d_6^*+d_7'^*) + (d_1+3d_2+3d_3+d_4+3d_9'+d_{10}')(d_5^*+d_8'^*)\} . \quad (45)$$

Where $d_1,...,d_6,d_7',...,d_{10}'$ defined by (26) and e_q is the quark electric charge and is given by

$$e_{u,c,t} = 2/3 , \qquad e_{d,s,b} = -(1/3) . \quad (46)$$

D) Effective Hamiltonian of Magnetic Dipole Decay Rates

We want to calculate the decay rates of $b \to q_i q_k \bar{q}_j$ according to the effective Hamiltonian ($Q_1,...,Q_6$), including magnetic dipole (Q_8) terms. We obtained the amplitude of operators $Q_1,...,Q_6$, and the amplitude of magnetic dipole (Q_8) terms. After that, to add the amplitudes and to calculate the decay rates of operators of the effective Hamiltonian including magnetic dipole terms ($Q_1,...,Q_6,Q_8$). The amplitude of the effective Hamiltonian including magnetic dipole is given by (see App.C)

$$\left|M^{tot}\right|^2_{spin-ave} = \frac{1}{4}[g_1 - 2v_k v_i (g_4-g_5)\cos(\theta_k-\theta_i) + g_2 + 2v_j v_k (g_6+g_7)\cos(\theta_j-\theta_k) - 2\sqrt{1-v_i^2}\sqrt{1-v_j^2}\, g_3 - 2v_j v_i (g_8-g_9)\cos(\theta_j-\theta_i)] , \quad (47)$$

For checking this equation we can to obtain the amplitude of tree-level and the effective Hamiltonian $(Q_1,...,Q_6)$. The amplitude of tree-level $(d_2=d_3=d_4=d_5=d_6=d_8=0)$ is given by

$$\left|M^{tot}\right|^2_{spin-ave,TL} = \frac{1}{4}[2h_1^2 - 2.h_1^2 v_i v_k \cos(\theta_k-\theta_i) + 0 + 0] = 3d_1^2 . \frac{1}{2}(1 - v_i v_k \cos(\theta_k-\theta_i)) . \quad (48)$$

and the amplitude of the effective Hamiltonian ($d_8=0$) is given by

$$\left|M^{tot}\right|^2_{spin-ave,EH} = (h_1^2/2)[1 - v_k v_i \cos(\theta_k-\theta_i)] + (h_3^2/2)[1 + v_j v_k \cos(\theta_j-\theta_k)] - (\sqrt{1-v_i^2}\sqrt{1-v_j^2}/4)(2h_1 h_3) . \quad (49)$$

The differential of decay rate of the effective Hamiltonian plus Magnetic Dipole (47) is given by

$$\frac{d^2\Gamma}{dxdy} = \frac{G_F^2 M_b^2}{192\pi^3}\frac{1}{2}(I_1+I_2+I_3) , \quad (50)$$

Where

$$I_1 = 6xyf_{ab}\ [g_1 - 2(g_4-g_5)h_{abc}] ,$$
$$I_2 = 6xyf_{bc}\ [g_2 + 2(g_6+g_7)h_{bca}] ,$$
$$I_3 = 6xyf_{ac}\ [-2g_3 h_{xa} h_{yc} - 2(g_8-g_9)h_{acb}] \quad (51)$$

and $f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{acb}, h_{xa}, h_{yc}$ defined by (B-

16). Now we want to check the decay rates of (50) for Tree-Level and effective Hamiltonian $(Q_1,...,Q_6)$. Putting $d_2 = d_3 = d_4 = d_5 = d_6 = d_8 = 0$ in (50) we obtained the decay rate of Tree-Level that is the same (43) and putting $d_8 = 0$ in (33) we obtained the decay rate of the effective Hamiltonian that is the same (39).

E) Effective Hamiltonian of Electroweak Penguin and Magnetic Dipole Decay Rates

The partial decay rate of the effective Hamiltonian plus Magnetic Dipole was given by Eq.(50). Now we want to obtain the partial decay rate including electroweak penguin. In this case, we must to include electroweak penguin operators $(d'_7, d'_8, d'_9, d'_{10})$ and the same way before; the partial decay rate is given by

$$\frac{d^2\Gamma}{dxdy} = \frac{G_F^2 M_b^2}{192\pi^3}\frac{1}{2}(I_1 + I_2 + I_3)\,, \tag{52}$$

Where I_1, I_2, I_3 defined by (51) and h_1, h_2, h_3 defined by,

$$h_1 = \sqrt{|d_1+d_2+d_3+d_4+d'_9+d'_{10}|^2+2|d_1+d_4+d'_{10}|^2+2|d_2+d_3+d'_9|^2}$$

$$h_2 = A_8 d_8\,,$$

$$h_3 = \sqrt{|d_5+d_6+d'_7+d'_8|^2+2|d_5+d'_8|^2+2|d_6+d'_7|^2}\,, \tag{53}$$

Results

As an example of the use of the above formalism, we use the standard Particle Data Group [45] parameterization of the CKM matrix with the central values

$\theta_{12} = 0.221$, $\theta_{13} = 0.0035$, $\theta_{23} = 0.041$,

and choose the CKM phase δ_{13} to be $\pi/2$. Following Ali and Greub [42], we treat internal quark masses in tree-level loops with the values (GeV) $m_b = 4.88$, $m_s = 0.2$, $m_d = 0.01$, $m_u = 0.005$, $m_c = 1.5$, $m_e = 0.0005$, $m_\mu = 0.1$, $m_\tau = 1.777$ and $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$. The effective Wilson coefficients C_i^{eff} at the renormalization scale $\mu = 2.5GeV$ for the various $b \to q$ $(\bar{b} \to \bar{q})$ transitions, shown in the Table 1 [33]. Also, following H.Y.Cheng [46], [47,31,36] and [48,49,31], we choose the effective Wilson coefficients of $C_7^{eff} - C_{10}^{eff}$,

$C_7^{eff} = -(0.0276 + i\,0.0369)\alpha_e$, $C_8^{eff} = 0.054\alpha_e$,

$C_9^{eff} = -(1.138 + i\,0.0369)\alpha_e$, $C_{10}^{eff} = 0.263\alpha_e$.

Here $\alpha_e = 1/137$, that is the electromagnetic coupling constant. We want to sum over the b-quark decay rates, to obtain the total rates at the tree-level. The total decay rate and branching ratios of several of semileptonic and hadronic modes show in Table 2. We see that modes $b \to cl\nu, b \to cd\bar{u}$ and $b \to cs\bar{c}$ are dominant. The total b-quark decay rate at the tree-level is given by

$$\Gamma^T_{total} = \Gamma^T_{Semileptonic} + \Gamma^T_{Hadronic} = 3.0457 \times 10^{-13} GeV\ .$$

We see that the decay rate for the antiparticle $\bar{b} \to \bar{u}\bar{d}u$ is greater than the decay rate particle $b \to ud\bar{u}$, and the decay rate antiparticle $\bar{b} \to \bar{c}\bar{d}c$ is less than the decay rate particle $b \to cd\bar{c}$, and so one. We consider that the modes $b \to c\bar{u}d$, and $b \to c\bar{c}s$ are dominant.

Table 1. Effective Wilson coefficients C_i^{eff} at the renormalization scale $\mu = 2.5$ for the various $b \to q$ $(\bar{b} \to \bar{q})$ transitions

	$b \to d$	$\bar{b} \to \bar{d}$	$b \to s$	$\bar{b} \to \bar{s}$
C_1^{eff}	1.1679+0.0000i	1.1679+0.0000i	1.1679+0.0000i	1.1679+0.0000i
C_2^{eff}	-0.3525+0.0000i	-0.3525+0.0000i	-0.3525+0.0000i	-0.3525+0.0000i
C_3^{eff}	0.0217+0.0018i	0.0234+0.0047i	0.0232+0.0030i	0.0231+0.0029i
C_4^{eff}	-0.0498-0.0054i	-0.0543-0.0142i	-0.0535-0.0091i	-0.0531-0.0086i
C_5^{eff}	0.0156+0.0018i	0.0173+0.0047i	0.0171+0.0030i	0.0170+0.0029i
C_6^{eff}	-0.0625-0.0054i	-0.0678-0.0142i	-0.0670-0.0091i	-0.0667-0.0086i

The branching ratios of the effective Hamiltonian $(Q_1,...,Q_6)$ for particles and antiparticles are collected in Table 3. In this case modes $b \to cd\bar{u}$ and $b \to cs\bar{c}$ are dominant. Also, the branching ratios including of the electroweak Penguin $(Q_1,...,Q_6,Q'_7,...,Q'_{10})$ show in Table 3 as well and the branching ratios of the pure Penguin show in Table 4. We consider that, in the pure Penguin decays, modes $b \to ss\bar{s}$ and $b \to sd\bar{d}$ are dominant, respectively. Also we see that, terms of Current-Current plus Penguin operators dominate as compared with the electroweak Penguin operators.

The branching ratios of the effective Hamiltonian plus Magnetic Dipole, and the effective Hamiltonian plus electroweak Penguin plus Magnetic dipole are show in Table 3 as well. We see that the electroweak Penguin plus Magnetic Dipole term is small and we can neglect this term in the total decay rate. The total decay rate of the effective Hamiltonian $(Q_1,...,Q_6)$ of particle and antiparticle is given by

$$\Gamma^{EH}_{total\,(Q_1,...,Q_6)} = 3.404\times10^{-13} GeV \ .$$

In addition, we can obtain the total decay rate of particles and antiparticles including electroweak penguin $(Q_1,...,Q_6,Q'_7,...,Q'_{10})$ for b-quark decays is given by

$$\Gamma^{EH+EP}_{total\,(Q_1,...,Q_{10})} = 3.497\times10^{-13} GeV \ .$$

Also the total decay rate of particles and antiparticles including effective Hamiltonian and Magnetic Dipole $(Q_1,...,Q_6,Q_8)$ and the effective Hamiltonian, electroweak penguin and Magnetic Dipole $(Q_1,...,Q_6,Q_8,Q'_7,...,Q'_{10})$ for b-quark decays is given by

$$\Gamma^{EH+MD}_{total\,(Q_1,...,Q_8)} = 3.526\times10^{-13} GeV \ .$$

$$\Gamma^{EH+EP+MD}_{total\,(Q_1,...,Q_8,...,Q_{10})} = 3.637\times10^{-13} GeV \ .$$

The total decay rates of pure penguin mode particles and antiparticles is

$$\Gamma^{P}_{total\,(Q_3,...,Q_6)} = 2.479\times10^{-15} GeV \ .$$

We see that for pure penguin modes, the decay rates of particles are less than the decay rates of antiparticles (see Table 4).

It is interesting if we compare the decay rate of the Tree-Level (T) (see (43)), effective Hamiltonian $(Q_1,...,Q_6)$ (EH) (see (39)), effective Hamiltonian plus electroweak Penguin $(Q_1,...,Q_6,Q'_7,...,Q'_{10})$ $(EH+EP)$ (see (45)),), effective Hamiltonian plus Magnetic Dipole $(EH+MD)$ (see (50)) and the effective Hamiltonian plus electroweak Penguin plus Magnetic Dipole $(Q_1,...,Q_6,Q_8,Q'_7,...,Q'_{10})$ $(EH+EP+MD)$ (see (52)) that show at Table 3.

Discussion

We obtained the decay rates of the b-quark at the tree-level, penguin, effective Hamiltonian, effective Hamiltonian including Electroweak Penguin, and for the first time, the effective Hamiltonian including Magnetic Dipole and the effective Hamiltonian including electroweak Penguin and Magnetic Dipole terms of the particles and antiparticles for the various $b \to q$ $(\bar{b} \to \bar{q})$ transitions. According to Table 2, the dominant mode in b-quark in the semileptonic and hadronic decays are, $b \to c\ell^-\bar{\nu}_\ell$ $(\ell \to e,\mu)$ and $b \to cd\bar{u}$ respectively because the decay rates of $b \to c$ channel are very much bigger than $b \to u$, since $V_{cb} >> V_{ub}$. In addition, the dominant mode in the pure penguin decays is, $b \to s$. According to Table 5, the branching ratios of pure penguin of the effective Hamiltonian of the particles and antiparticles are close.

The electroweak penguin and magnetic dipole terms are small for b-quark decay rates (electroweak corrections and the magnetic dipole contributions are small) and the decay rate of the tree, effective Hamiltonian, effective Hamiltonian including Electroweak Penguin, effective Hamiltonian including Magnetic Dipole and the effective Hamiltonian including electroweak Penguin and Magnetic Dipole of the particles and antiparticles are also not very different (see Table 3). The decay rates of $b-$ and $\bar{b}-$quark, at the tree-level are exactly the same, but in the pure

Table 2. Branching ratios (BR) of tree-level $b \to q_i q_k \bar{q}_j$ (43) ($\Gamma_{tot} = 3.0457\times10^{-13} GeV$)

Process	$BR\times10^{-2}$	Process	$BR\times10^{-2}$
$b \to ce^-\bar{\nu}_e$	14.62	$b \to ue^-\bar{\nu}_e$	0.231
$b \to c\mu^-\bar{\nu}_\mu$	14.62	$b \to u\mu^-\bar{\nu}_\mu$	0.231
$b \to c\tau^-\bar{\nu}_\tau$	0.714	$b \to u\tau^-\bar{\nu}_\tau$	0.084
$b \to cd\bar{u}$	49.02	$b \to ud\bar{u}$	0.725
$b \to cs\bar{c}$	16.13	$b \to ud\bar{c}$	0.019
$b \to cd\bar{c}$	0.857	$b \to us\bar{u}$	0.531
$b \to cs\bar{u}$	2.352	$b \to us\bar{c}$	0.355

Table 3. Branching ratios ($BR\times10^{-2}$) of tree-level (T) (43), Effective Hamiltonian (EH) (39), Effective Hamiltonian including Electroweak Penguin $(EH+EP)$ (45), Effective Hamiltonian including Magnetic Dipole $(EH+MD)$ (50) and Effective Hamiltonian including Electroweak Penguin and Magnetic Dipole $(EH+EP+MD)$ (52) of the particles and antiparticles for the various $b\to q$ $(\bar{b}\to\bar{q})$ transitions. The total decay rates are in unit of $10^{-13}GeV$

$\Gamma_{tot}\to$	3.0457	3.404	3.497	3.529	3.637
Process	**(*T*)**	**(*E–H*)**	**(*EH+EP*)**	**(*EH+MD*)**	**(*EH+EP+MD*)**
$b\to ud\bar{u}$	0.725	0.899	0.872	0.903	0.914
$b\to cd\bar{c}$	0.857	0.995	0.956	0.992	1.102
$b\to cd\bar{u}$	49.02	49.83	48.51	50.35	52.26
$b\to ud\bar{c}$	0.019	0.024	0.024	0.019	0.038
$b\to us\bar{u}$	0.531	0.603	0.544	0.680	0.714
$b\to cs\bar{c}$	16.13	17.09	18.19	17.780	18.69
$b\to us\bar{c}$	0.355	0.658	0.578	0.697	0.746
$b\to cs\bar{u}$	2.352	2.623	3.716	0.521	3.962
$\bar{b}\to\bar{u}\bar{d}u$	0.725	0.943	0.915	0.951	0.988
$\bar{b}\to\bar{c}\bar{d}c$	0.857	0.987	0.958	1.124	1.658
$\bar{b}\to\bar{c}\bar{d}u$	49.02	49.80	48.49	50.31	50.87
$\bar{b}\to\bar{u}\bar{d}c$	0.019	0.023	0.022	0.032	0.045
$\bar{b}\to\bar{u}\bar{s}u$	0.531	0.595	0.539	0.675	0.721
$\bar{b}\to\bar{c}\bar{s}c$	16.13	17.10	18.18	17.791	18.38
$\bar{b}\to\bar{u}\bar{s}c$	0.355	0.655	0.577	0.694	0.762
$\bar{b}\to\bar{c}\bar{s}u$	2.352	2.623	3.707	2.512	4.130

penguin, effective Hamiltonian, effective Hamiltonian including electroweak Penguin, effective Hamiltonian including Magnetic Dipole and the effective Hamiltonian including electroweak Penguin and Magnetic Dipole, they are different. For example, $\Gamma_{b\to sd\bar{d}}<\Gamma_{\bar{b}\to\bar{s}\bar{d}d}$, $\Gamma_{b\to ud\bar{u}}<\Gamma_{\bar{b}\to\bar{u}\bar{d}u}$, $\Gamma_{b\to cd\bar{u}}>\Gamma_{\bar{b}\to\bar{c}\bar{d}u}$ and $\Gamma_{b\to cs\bar{c}}\approx\Gamma_{\bar{b}\to\bar{c}\bar{s}c}$, because the total decay rates of $b-$ and $\bar{b}-$ quark must be equal, $\Gamma_b^{total}=\Gamma_{\bar{b}}^{total}$.

Also the decay rates and branching ratios are very similar in all the models but the effective Hamiltonian including electroweak Penguin and Magnetic Dipole total decay rate is about 10% larger than the simple tree or effective Hamiltonian. On the other hand, including the penguin induces matter antimatter asymmetries. These are largest in the rare decays $b\to ud\bar{u}$, the decay rate of which, is about 7% smaller than the decay rate $\bar{b}\to\bar{u}\bar{d}u$. Also the rate $b\to su\bar{u}$ is larger than the rate $\bar{b}\to\bar{s}\bar{u}u$.

Table 4. Branching Ratios of pure penguin of Effective Hamiltonian of the particles and antiparticles for the various $b\to q$ $(\bar{b}\to\bar{q})$ transitions (44) ($\Gamma_{tot}=3.404\times10^{-13}GeV$)

Process	$BR\times10^{-4}$	Process	$BR\times10^{-4}$
$b\to dd\bar{d}$	2.408	$\bar{b}\to\bar{d}\bar{d}d$	2.615
$b\to ds\bar{s}$	2.687	$\bar{b}\to\bar{d}\bar{s}s$	2.763
$b\to sd\bar{d}$	53.872	$\bar{b}\to\bar{s}\bar{d}d$	54.053
$b\to ss\bar{s}$	54.517	$\bar{b}\to\bar{s}\bar{s}s$	53.782

Appendix A: Penguin Amplitude of Magnetic Dipole

According to (4) the penguin amplitude is given by

$$M^{dip}=\frac{g_s^2}{4\pi^2}[\bar{u}_k(p_k)T^a(i\sigma_{\mu\nu}q^\nu/q^2)\\ F_2^R(0)P_Ru_b(p_b)][\bar{u}_i(p_i)\gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})]. \quad \text{(A-1)}$$

Where

$$\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu] = (i/2)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) . \quad \text{(A-2)}$$

And

$$\gamma^\mu\gamma^\nu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^\nu \\ \tilde{\sigma}^\nu & 0 \end{pmatrix} = \begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu \end{pmatrix}. \quad \text{(A-3)}$$

So

$$\sigma^{\mu\nu} = \frac{i}{2}\begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu - \sigma^\nu\tilde{\sigma}^\mu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu - \tilde{\sigma}^\nu\sigma^\mu \end{pmatrix}. \quad \text{(A-5)}$$

The wave functions of b and q_k are given by

$$\bar{u}_k \sigma_{\mu\nu}[(1+\gamma_5)/2]u_b$$
$$= \frac{i}{2}\begin{pmatrix} \psi_{kL} \\ \psi_{kR} \end{pmatrix}^\dagger \begin{pmatrix} \tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu & 0 \\ 0 & \sigma_\mu\tilde{\sigma}_\nu - \sigma_\nu\tilde{\sigma}_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \psi_{bR} \end{pmatrix}$$
$$= (i/2)\psi_{kL}(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)\psi_{bR} . \quad \text{(A-6)}$$

Putting in the penguin amplitude

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)[\bar{u}_{kL}\frac{q^\nu}{q^2}(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)u_{bR}][\bar{u}_i(\tilde{\sigma}^\mu + \sigma^\mu)v_{\bar{j}}]. \quad \text{(A-7)}$$

Putting $q^\nu = (p_b - p_k)^\nu$ in the above equation,

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[(\bar{u}_{kL}\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)u_{bR}) - (\bar{u}_{kL}(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu u_{bR})]$$
$$\times[\bar{u}_i\tilde{\sigma}^\mu v_{\bar{j}} + \bar{u}_i\sigma^\mu v_{\bar{j}}] . \quad \text{(A-8)}$$

or

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[\langle k_L|\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)|b_R\rangle - \langle k_L|(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu|b_R\rangle]$$
$$\times[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle] . \quad \text{(A-9)}$$

We known that,

$$\tilde{\sigma}_\mu p_b^\mu|b_L\rangle = m_b|b_R\rangle, \quad \tilde{\sigma}_\mu p_k^\mu|k_L\rangle = m_k|k_R\rangle,$$
$$\sigma_\nu p_b^\nu|b_R\rangle = m_b|b_L\rangle, \quad \sigma_\nu p_k^\nu|k_R\rangle = m_k|k_L\rangle,$$
$$\tilde{\sigma}_\mu(\sigma_\nu p_k^\nu) = p_{k\mu}, \quad (\tilde{\sigma}_\mu p_b^\nu)\sigma_\mu = p_{b\mu} . \quad \text{(A-10)}$$

And

$$(p_b + p_k)_\mu = (2p_b - p_i - p_j)_\mu . \quad \text{(A-11)}$$

Also according to conservation of current

$$(p_i + p_j)_\mu[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle]$$
$$= m_i[\langle i_R|j_L\rangle + \langle i_L|j_R\rangle] - m_j[\langle i_L|j_R\rangle + \langle i_R|j_L\rangle] = 0 , \quad \text{(A-12)}$$

Since in the penguin decays is $m_i = m_j$ and $|j\rangle$ is the antiparticle, so

$$\tilde{\sigma}_\mu p_{j\mu}|j_L\rangle = -m_j|j_R\rangle . \quad \text{(A-13)}$$

Consequently, the magnetic dipole term of penguin amplitude becomes to

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle + m_k\langle k_R|\sigma_\mu|b_R\rangle - (p_b + p_k)_\mu\langle k_L|b_R\rangle]$$
$$\times[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle] . \quad \text{(A-14)}$$

We neglected from term $m_k\langle k_R|\sigma_\mu|b_R\rangle$ because $m_k << m_b$, so

$$M^{dip} = (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle$$
$$-(p_b + p_k)_\mu\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - (p_b + p_k)_\mu\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle] . \quad \text{(A-15)}$$

Using (A-12) for the second part of equation above, thus

$$M^{dip} = (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle$$
$$-2p_{b\mu}\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - 2p_{b\mu}\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle] . \quad \text{(A-16)}$$

Here b meson is at the rest ($p_{b\mu} = (m_b, \vec{0})$) and

$$(T^aT^a) = 4/3,$$

$$d_8 = -\frac{g_s^2}{8\pi^2} F_2^R(0) = -\frac{g_s^2}{8\pi^2} m_b \frac{g_2^2}{8M_W^2} \sum_i V_{ik}^* V_{ib} f_2(x_i) ,$$

$$= -(2\sqrt{2}G_F)(m_b/2)(\alpha_s/4\pi)\sum_i V_{ik}^* V_{ib} f_2(x_i) . \quad \text{(A-17)}$$

So, the magnetic dipole of penguin amplitude is given by

$$M^{dip} = (4/3)d_8(m_b/q^2)[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle$$
$$+\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle - 2\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle$$
$$-2\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle] . \quad \text{(A-18)}$$

The b quark is at the rest and to have spin projection –1/2 along angle θ_b, thus the spin projection of b quark of +1/2 is along $\theta_b - \pi$ ($\theta_b \to \theta_b - \pi$),

$$b\ spin(-1/2)\ and\ angle\ \theta_b$$
$$\propto (1/\sqrt{2})\begin{pmatrix} -\sin(\theta_b/2) \\ \cos(\theta_b/2) \end{pmatrix},$$

$$b\ spin(+1/2)\ and\ angle\ \theta_b$$
$$\propto (1/\sqrt{2})\begin{pmatrix} \cos(\theta_b/2) \\ \sin(\theta_b/2) \end{pmatrix}. \quad \text{(A-19)}$$

Putting the factor of $(1/\sqrt{2})$ in the M^{dip} and negligible terms $\langle k_L|b_R\rangle$, thus the amplitude of magnetic dipole becomes to

$$M^{dip} = A_8 d_8[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \quad \text{(A-20)}$$

Here

$$A_8 = (1/\sqrt{2})(4/3)(m_b/q^2) . \quad \text{(A-21)}$$

Terms $(\tilde{\sigma}_\mu)(\tilde{\sigma}^\mu)$ and $(\tilde{\sigma}_\mu)(\sigma^\mu)_{LR}$ for spin +1/2 and –1/2 obtain by the matrix elements of $L-L$ handed and $L-R$ handed for the b quark

$$\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(1/2)}\rangle_L\,\langle -_k|_L\,\tilde{\sigma}_\mu|-_j\rangle_L$$
$$= \sin((\theta_k - \theta_j - \theta_i)/2) + \sin((\theta_k + \theta_j - \theta_i)/2),$$
$$\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(-1/2)}\rangle_L\,\langle -_k|_L\,\tilde{\sigma}_\mu|-_j\rangle_L$$
$$= \cos((\theta_k - \theta_j - \theta_i)/2) - \cos((\theta_k + \theta_j - \theta_i)/2). \quad \text{(A-22)}$$

When dealing with penguin amplitudes we will also need the matrix elements

$$\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(1/2)}\rangle_L\,\langle -_k|_R\,\sigma_\mu|-_j\rangle_R =$$
$$\sin((\theta_i - \theta_k - \theta_j)/2) - \sin((\theta_i + \theta_k - \theta_j)/2) ,$$
$$\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(-1/2)}\rangle_L\,\langle -_k|_R\,\sigma_\mu|-_j\rangle_R =$$
$$\cos((\theta_i - \theta_k - \theta_j)/2) + \cos((\theta_i + \theta_k - \theta_j)/2) . \quad \text{(A-23)}$$

The first term of (A-20) for b spin project –1/2, according to Fierz transformation,

$$\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle$$
$$= -\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle . \quad \text{(A-24)}$$

is given by

$$M_{1(-1/2)}^{dip} \equiv -A_8 d_8\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle ,$$
$$= A_8 d_8[-\cos((\theta_b - \theta_i - \theta_k + \theta_j)/2)$$
$$-\cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] . \quad \text{(A-25)}$$

And the first term for b spin project +1/2 is given by

$$M_{1(+1/2)}^{dip} \equiv -A_8 d_8\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle ,$$
$$= A_8 d_8[-\sin((\theta_b + \theta_i - \theta_k - \theta_j)/2)$$
$$+\sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] . \quad \text{(A-26)}$$

Also the second term of (A-20) for b spin project –1/2 is given by

$$M_{2(-1/2)}^{dip} \equiv -A_8 d_8\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_R|\sigma_\mu|j_R\rangle ,$$
$$= A_8 d_8[-\cos((\theta_b + \theta_i - \theta_k - \theta_j)/2)$$
$$-\cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] . \quad \text{(A-27)}$$

In addition the second term for b spin project +1/2 is given by

$$M_{2(+1/2)}^{dip} \equiv -A_8 d_8\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_R|\sigma_\mu|j_R\rangle ,$$
$$= A_8 d_8[-\sin((\theta_b - \theta_i - \theta_k + \theta_j)/2)$$
$$+\sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] . \quad \text{(A-28)}$$

So the penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by

$$M^{dip}_{(+1/2)} = A_8 d_8 \{[-\sin((\theta_b + \theta_i - \theta_k - \theta_j)/2)$$
$$+\sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)]$$
$$+[-\sin((\theta_b - \theta_i - \theta_k + \theta_j)/2)$$
$$+\sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)]\} \quad \text{(A-29)}$$

$$M^{dip}_{(-1/2)} = A_8 d_8 \{[-\cos((\theta_b - \theta_i - \theta_k + \theta_j)/2)$$
$$-\cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)]$$
$$+[-\cos((\theta_b + \theta_i - \theta_k - \theta_j)/2)$$
$$-\cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)]\} \quad \text{(A-30)}$$

Appendix B: Decay Rate of the Effective Hamiltonian

The effective $\Delta B = 1$ Hamiltonian at scale $\mu = \mathrm{O}(m_b)$ for tree plus penguin and including the electroweak penguin and the magnetic dipole term is

$$H^{\Delta B=1}_{eff} = 2\sqrt{2} G_F \{[d_{1c}(\mu) Q^c_1(\mu) + d_{2c}(\mu) Q^c_2(\mu)]$$
$$+[d_{1u}(\mu) Q^u_1(\mu) + d_{2u}(\mu) Q^u_2(\mu)]$$
$$-[\sum_{i=3}^{6,8} d_i(\mu) Q_i(\mu) + \sum_{i=7}^{10} d'_i(\mu) Q'_i(\mu)]\} \,. \quad \text{(B-1)}$$

Here $d_1,...,_6, d'_7,...,_{10}$ are defined by (26), $d_{1,2c,u} = d_{1,2}(i = j = c,u)$ and index k refer to d or s. According to (A-22) and (A-23) we can obtain the matrix elements of the effective Hamiltonian operators. In the first step, we choose tree plus penguin operators $(Q_1,...,Q_6)$. All of the terms Q_1, Q_2, Q_3, Q_4 have a form L-L handed but terms Q_1, Q_2 have a form $\langle i|_L \tilde{\sigma}^\mu |b\rangle_L \langle k|_L \tilde{\sigma}_\mu |j\rangle_L$ and terms Q_3, Q_4 have a form $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_L \tilde{\sigma}_\mu |j\rangle_L$. Consider i, k and j momenta in XZ plane, so according to (A-22), for the Q_1, Q_2 and Q_3, Q_4, we can write for b spin projection 1/2 and –1/2. Also terms Q_1, Q_2 and Q_3, Q_4 differ only by a minus, because

$$\sin((\theta_i + \theta_j - \theta_k)/2) = -\sin((\theta_k - \theta_j - \theta_i)/2) \,,$$
$$\sin((\theta_i - \theta_j - \theta_k)/2) = -\sin((\theta_k + \theta_j - \theta_i)/2) \,. \quad \text{(B-2)}$$

Terms Q_5, Q_6 are of the form L-R handed, $\langle i|_L \tilde{\sigma}^\mu |b\rangle_L \langle k|_R \sigma_\mu |j\rangle_R$. The main forms of the terms Q_5, Q_6 are $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_R \sigma_\mu |j\rangle_R$. We can write these terms, according to (A-23). So, the matrix element for b quark spin project 1/2 is given by

$$M_{eff} = 2\sqrt{2} G_F \{(A_1 + A_2)[\sin((\theta_k - \theta_j - \theta_i)/2)$$
$$+\sin((\theta_k + \theta_j - \theta_i/2)] - A_3[\sin((\theta_k - \theta_i - \theta_j)/2)$$
$$-\sin((\theta_k + \theta_i - \theta_j)/2)]\} \,. \quad \text{(B-3)}$$

Here A_1, A_2 and A_3 are combination of Wilson coefficients and colour factors. The forms of $(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL}$ and $(\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}$, are according to (A-22) and (A-23). So squaring spin average term $Q_1,...,Q_6$ is given by

$$[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]^2_{sp-av}$$
$$= \alpha_1 (1/16)(1+v_i)(1+v_k)(1+v_j)[1 - \cos(\theta_k - \theta_i)]$$
$$+\alpha_2 (1/16)(1-v_i)(1+v_k)(1-v_j)[1 + \cos(\theta_k - \theta_j)]$$
$$+\alpha_3 (1/16)\sqrt{1-v_i^2}(1+v_k)\sqrt{1-v_j^2}[1 + \cos(\theta_j - \theta_i)$$
$$-\cos(\theta_k - \theta_j) - \cos(\theta_k - \theta_i)] \,. \quad \text{(B-4)}$$

Now, we must obtain all of the helicity states for $Q_1,...,Q_6$ and then to be added. Adding eight terms of helicity states, so

$$[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]^2_{sp-av}$$
$$= (\alpha_1/2)[1 - v_i v_k \cos(\theta_k - \theta_i)]$$
$$+(\alpha_2/2)[1 + v_k v_j \cos(\theta_j - \theta_k)]$$
$$+(\alpha_3/2)\sqrt{1-v_i^2}\sqrt{1-v_j^2} \,. \quad \text{(B-5)}$$

After adding all colour combinations α_1, α_2 and α_3 gives

$$\alpha_1 = |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2 \,,$$
$$\alpha_2 = |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2) \,,$$
$$\alpha_3 = \mathrm{Re}\{(3d_1 + d_2 + d_3 + 3d_4) d_6^*$$
$$+(d_1 + 3d_2 + 3d_3 + d_4) d_5^*\} \,. \quad \text{(B-6)}$$

Here $d_1,...,d_6$ defined by (26). The energy conservation gives

$$\cos(\theta_k - \theta_i) = [(M_l - E_i - E_k)^2$$
$$-(m_j^2 + p_i^2 + p_k^2)]/2 p_i p_k \,,$$

$$\cos(\theta_j - \theta_k) = [(M_l - E_k - E_j)^2$$
$$-(m_i^2 + p_k^2 + p_j^2)] / 2p_k p_j . \quad \text{(B-7)}$$

The angle between the particle velocities must be physical, $-1 \le \cos(\theta_k - \theta_i) \le 1$ and $-1 \le \cos(\theta_j - \theta_k) \le 1$. So we should take the variable p_i and p_k, or x and y as,

$$p_i = xM_l / 2 , \qquad p_k = yM_l / 2 \quad \text{(B-8)}$$

Then p_j is given by energy conservation $E_j = M_b - E_i - E_k = \sqrt{m_j^2 + p_j^2}$. Also $\cos(\theta_i - \theta_k) = (p_j^2 - p_i^2 - p_k^2)/2p_i p_k$, and so on. Momentum conservation gives

$$p_k \cos(\theta_i - \theta_k) + p_j \cos(\theta_j - \theta_i) = -p_i , \quad \text{(B-9)}$$

and e.t.c. Also

$$p_k \sin(\theta_i - \theta_k) = \pm p_j \sin(\theta_j - \theta_i) . \quad \text{(B-10)}$$

and e.t.c. The partial decay rate, b spin averaged and summed over final spin states, has overall spherical symmetry. Apart from its overall orientation, a final state is specified by only two parameters, say $p_i = |\boldsymbol{p_i}|$ and $p_k = |\boldsymbol{p_k}|$. The partial decay rate in the b rest frame is

$$d^2\Gamma_{Q_1,\dots,Q_6} / dp_i dp_k$$
$$= (G_F^2 / \pi^3) p_i p_k E_{\bar{j}} \{\alpha_1 (p_i . p_k / E_i E_k)$$
$$+ \alpha_2 (p_i . p_{\bar{j}} / E_i E_{\bar{j}}) + \alpha_3 (m_k m_{\bar{j}} / E_k E_{\bar{j}})\} . \quad \text{(B-11)}$$

Here

$$p_i . p_k = (M_b^2 + m_{\bar{j}}^2 - m_i^2 - m_k^2 - 2M_b E_{\bar{j}}) / 2 ,$$
$$p_i . p_{\bar{j}} = (M_b^2 + m_k^2 - m_i^2 - m_{\bar{j}}^2 - 2M_b E_k) / 2 . \quad \text{(B-12)}$$

After the change of variable to x and y, the decay rate is given by

$$d^2\Gamma_{Q_1,\dots,Q_6} / dxdy = \Gamma_{0b} I_{ps}^{EH} . \quad \text{(B-13)}$$

$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3 . \quad \text{(B-14)}$$

where

$$I_{ps}^1 = 6xy . f_{ab} . (1 - h_{abc}) ,$$
$$I_{ps}^2 = 6xy . f_{bc} . (1 + h_{bca}) ,$$
$$I_{ps}^3 = 6xy . f_{ac} . h_{xa} . h_{yc} . \quad \text{(B-15)}$$

Here

$$f_{ab} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + b^2} ,$$
$$h_{abc} = \frac{(f_{ab})^2 - (c^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + b^2}} ,$$
$$f_{bc} = 2 - \sqrt{x^2 + b^2} - \sqrt{y^2 + c^2} ,$$
$$h_{bca} = \frac{(f_{bc})^2 - (a^2 + x^2 + y^2)}{2\sqrt{x^2 + b^2}\sqrt{y^2 + c^2}} ,$$
$$f_{ac} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + c^2} ,$$
$$h_{acb} = \frac{(f_{ac})^2 - (b^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + c^2}} .$$
$$h_{xa} = [1 - (x^2 / (x^2 + a^2))]^{1/2} ,$$
$$h_{yc} = [1 - (y^2 / (y^2 + c^2))]^{1/2} . \quad \text{(B-16)}$$

where a, b and c are:

$$a = 2m_i / M_b , \; b = 2m_k / M_b , \; c = 2m_j / M_b . \quad \text{(B-17)}$$

Appendix C: Effective Hamiltonian of Magnetic Dipole Decay Rate

We want to calculate the decay rates of $b \to q_i q_k \bar{q}_j$ according to Effective Hamiltonian ($Q_1,\dots,Q_6$), including magnetic dipole (Q_8) terms. The amplitude of Effective Hamiltonian for operators Q_1, Q_2, Q_3, Q_4 is given by

$$M\,1_{(1/2)}^{L-L} = A(d_1 + d_2 + d_3 + d_4)[\sin((\theta_k - \theta_j - \theta_i)/2)$$
$$+ \sin((\theta_k + \theta_j - \theta_i)/2)] ,$$
$$M\,2_{(-1/2)}^{L-L} = A(d_1 + d_2 + d_3 + d_4)[\cos((\theta_k - \theta_j - \theta_i)/2)$$
$$- \cos((\theta_k + \theta_j - \theta_i)/2)] , \quad \text{(C-1)}$$

And for operators Q_5, Q_6 is as well

$$M\,3_{(1/2)}^{L-R} = A'(d_5 + d_6)[\sin((\theta_k - \theta_i - \theta_j)/2)$$
$$- \sin((\theta_k + \theta_i - \theta_j)/2)] ,$$
$$M\,4_{(-1/2)}^{L-R} = A'(d_5 + d_6)[\cos((\theta_k - \theta_i - \theta_j)/2)$$

$$+\cos((\theta_k+\theta_i-\theta_j)/2)],\qquad \text{(C-2)}$$

Where $d_1,...,d_6,d_8$ and A_8 defined by (26) and (A-21) respectively and

$$A=\sqrt{(1+v_i)}\sqrt{(1+v_k)}\sqrt{(1+v_{\bar{j}})}\ ,$$

$$A'=\sqrt{(1-v_i)}\sqrt{(1+v_k)}\sqrt{(1-v_{\bar{j}})}\ ,\qquad \text{(C-3)}$$

Also the amplitude of magnetic dipole according to (A-20) is given by

$$M^{dip}=A_8d_8[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle$$

$$+\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle],\qquad \text{(C-4)}$$

For various b spin project 1/2 and –1/2 the first term of (C-4) is given by

$$M\,5^{dip,L-L}_{(1/2)}=AA_8d_8[\sin((\theta_i-\theta_k+\theta_j)/2),$$

$$+\sin((\theta_i+\theta_k-\theta_j)/2)]$$

$$M\,6^{dip,L-L}_{(-1/2)}=AA_8d_8[-\cos((\theta_i-\theta_k+\theta_j)/2)$$

$$-\cos((\theta_i+\theta_k-\theta_j)/2)],\qquad \text{(C-5)}$$

And for the second term of (C-4) is given by

$$M\,7^{dip,L-R}_{(1/2)}=A'A_8d_8[\sin((\theta_i-\theta_k+\theta_j)/2)$$

$$-\sin((\theta_i-\theta_k-\theta_j)/2)],$$

$$M\,8^{dip,L-R}_{(-1/2)}=A'A_8d_8[-\cos((\theta_i-\theta_k+\theta_j)/2)$$

$$-\cos((\theta_i-\theta_k-\theta_j)/2)],\qquad \text{(C-6)}$$

Now we must add the amplitude of $Q_1,...,Q_6$ and Q_8 (magnetic dipole term) for b spine project 1/2 and –1/2, so

$$M^{tot}_{(1/2)}=M\,1^{L-L}_{(1/2)}+M\,3^{L-R}_{(1/2)}+M\,5^{dip,L-L}_{(1/2)}+M\,7^{dip,L-R}_{(1/2)},$$

$$M^{tot}_{(-1/2)}=M\,2^{L-L}_{(-1/2)}+M\,4^{L-R}_{(-1/2)}$$

$$+M\,6^{dip,L-L}_{(-1/2)}+M\,8^{dip,L-R}_{(-1/2)},\qquad \text{(C-7)}$$

or

$$M^{tot}_{(-1/2)}=e_1\cos((\theta_k-\theta_i-\theta_j)/2)$$

$$-e_2\cos((\theta_k-\theta_i+\theta_j)/2)+e_3\cos((\theta_k+\theta_i-\theta_j)/2)$$

$$M^{tot}_{(1/2)}=e_1\sin((\theta_k-\theta_i-\theta_j)/2)$$

$$+e_2\sin((\theta_k-\theta_i+\theta_j)/2)$$

$$-e_3\sin((\theta_k+\theta_i-\theta_j)/2)\qquad \text{(C-8)}$$

Here

$$e_1=A[(d_1+d_2+d_3+d_4)+A_8d_8]$$

$$+A'\,[(d_5+d_6)+A_8d_8],$$

$$e_2=A(d_1+d_2+d_3+d_4)-A'\,A_8d_8,$$

$$e_3=AA_8d_8+A'\,(d_5+d_6),\qquad \text{(C-9)}$$

The spin average of b spin project of 1/2 and –1/2 is given by

$$\left|M^{tot}\right|^2_{spin-ave}=\frac{1}{2}[\left|M^{tot}_{(1/2)}\right|^2+\left|M^{tot}_{(-1/2)}\right|^2]$$

$$=\frac{1}{2}[e_1^2+e_2^2+e_3^2-2e_1e_2\cos(\theta_k-\theta_i)$$

$$+2e_1e_3\cos(\theta_j-\theta_k)-2e_2e_3\cos(\theta_j-\theta_i)]\qquad \text{(C-10)}$$

After adding all color factors gives

$$e_1=A[h_1+h_2]+A'\,[h_2+h_3],$$

$$e_2=Ah_1-A'\,h_2,$$

$$e_3=Ah_2+A'\,h_3,\qquad \text{(C-11)}$$

Where

$$h_1=\sqrt{|d_1+d_2+d_3+d_4|^2+2|d_1+d_4|^2+2|d_2+d_3|^2}\ ,$$

$$h_2=A_8d_8,\ h_3=\sqrt{|d_5+d_6|^2+2|d_5|^2+2|d_6|^2}\ ,\qquad \text{(C-12)}$$

The first term of (C-10) is given by

1) $e_1^2+e_2^2+e_3^2=A^2(2h_1^2+2h_2^2+2h_1h_2)$

$$+A'(2h_2^2+2h_3^2+2h_2h_3)$$

$$+2AA'(h_2^2+h_1h_3+2h_2h_3)\qquad \text{(C-13)}$$

Adding eight terms of helicity states, so

$$e_1^2+e_2^2+e_3^2=8(g_1+g_2+2\sqrt{1-v_i^2}\sqrt{1-v_j^2}\ g_3),\qquad \text{(C-14)}$$

The second term of (C-10) is given by

2) $2e_1e_2\cos(\theta_k-\theta_i)=2\{\ A^2(h_1^2+h_1h_2)$

$$-A'^2(h_2^2+h_2h_3)$$

$$+AA'(h_1h_3-h_2^2)\ \}\cos(\theta_k-\theta_i),\qquad \text{(C-15)}$$

Adding eight terms of helicity states, so

$$2e_1e_2\cos(\theta_k-\theta_i)$$

$$=2.8v_kv_i(g_4-g_5)\cos(\theta_k-\theta_i), \quad \text{(C-16)}$$

The third term of (C-10) is given by

3) $2e_1e_3\cos(\theta_j-\theta_k)=2\{\ A^2(h_2^2+h_1h_2)$

$+A'^2(h_3^2+h_2h_3)$

$+AA'(h_1h_3+2h_2h_3+h_2^2)\ \}\cos(\theta_j-\theta_k)$ (C-17)

Adding eight terms of helicity states, so

$$2e_1e_3\cos(\theta_j-\theta_k)$$

$$=2.8v_jv_k(g_6+g_7)\cos(\theta_j-\theta_k), \quad \text{(C-18)}$$

The fourth term of (C-10) is given by

4) $2e_2e_3\cos(\theta_j-\theta_i)=2\{A^2(h_1h_2)-A'^2(h_2h_3)$

$+AA'(h_1h_3-h_2^2)\ \}\cos(\theta_j-\theta_i)$, (C-19)

Adding eight terms of helicity states, so

$$2e_2e_3\cos(\theta_j-\theta_i)$$

$$=2.8v_jv_i(g_8-g_9)\cos(\theta_j-\theta_i), \quad \text{(C-20)}$$

Here

$g_1=2h_1^2+2h_2^2+2h_1h_2$,

$g_2=2h_2^2+2h_3^2+2h_2h_3$,

$g_3=h_2^2-h_1h_2+h_1h_3+3h_2h_3$,

$g_4=h_1^2+h_1h_2$,

$g_5=h_2^2+h_2h_3$,

$g_6=h_2^2+h_1h_2$,

$g_7=h_3^2+h_2h_3$,

$g_8=h_1h_2$,

$g_9=h_2h_3$, (C-21)

The total amplitude of (C-10) is given by

$$\left|M^{tot}\right|^2_{spin-ave}=\frac{1}{4}[g_1+g_2-2\sqrt{1-v_i^2}\sqrt{1-v_j^2}g_3$$

$$-2v_kv_i(g_4-g_5)\cos(\theta_k-\theta_i)$$

$$+2v_jv_k(g_6+g_7)\cos(\theta_j-\theta_k)$$

$$-2v_jv_i(g_8-g_9)\cos(\theta_j-\theta_i)], \quad \text{(C-22)}$$

or

$$\left|M^{tot}\right|^2_{spin-ave}=\frac{1}{4}[g_1-2v_kv_i(g_4-g_5)\cos(\theta_k-\theta_i)$$

$$+g_2+2v_jv_k(g_6+g_7)\cos(\theta_j-\theta_k)$$

$$-2\sqrt{1-v_i^2}\sqrt{1-v_j^2}g_3$$

$$-2v_jv_i(g_8-g_9)\cos(\theta_j-\theta_i)], \quad \text{(C-23)}$$

Also we can to obtain the amplitude of tree-level and Effective Hamiltonian $(Q_1,...,Q_6)$. The amplitude of tree-level ($d_2=d_3=d_4=d_5=d_6=d_8=0$) is given by

$$\left|M^{tot}\right|^2_{spin-ave,TL}=\frac{1}{4}[2h_1^2-2.h_1^2v_iv_k\cos(\theta_k-\theta_i)+0+0]$$

$$=3d_1^2.\frac{1}{2}(1-v_iv_k\cos(\theta_k-\theta_i)). \quad \text{(C-24)}$$

and the amplitude of Effective Hamiltonian ($d_8=0$) is given by

$$\left|M^{tot}\right|^2_{spin-ave,EH}=(h_1^2/2)[1-v_kv_i\cos(\theta_k-\theta_i)]$$

$$+(h_3^2/2)[1+v_jv_k\cos(\theta_j-\theta_k)]$$

$$-(\sqrt{1-v_i^2}\sqrt{1-v_j^2}/4)(2h_1h_3). \quad \text{(C-25)}$$

After integration in the phase space and change variable x and y,

$$x=2p_i/m_b,\ y=2p_k/m_b, \quad \text{(C-26)}$$

The differential of decay rate of Effective Hamiltonian plus Magnetic Dipole (C-23) is given by

$$\frac{d^2\Gamma}{dxdy}=\frac{G_F^2M_b^2}{192\pi^3}\frac{1}{2}(I_1+I_2+I_3), \quad \text{(C-27)}$$

Where

$I_1=6xyf_{ab}\ [g_1-2(g_4-g_5)h_{abc}]$,

$I_2=6xyf_{bc}\ [g_2+2(g_6+g_7)h_{bca}]$,

$I_3=6xyf_{ac}\ [-2g_3h_{xa}h_{yc}-2(g_8-g_9)h_{acb}]$ (C-28)

and $f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{acb}, h_{xa}, h_{yc}$ defined by (B-16).

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