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Entanglement of an Atom and Its Spontaneous Emission Fields via Spontaneously Generated Coherence

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Abstract

The entanglement between a Λ-type three-level atom and its spontaneous emission fields is investigated. The effect of spontaneously generated coherence (SGC) on entanglement between the atom and its spontaneous emission fields is then discussed. We find that in the presence of SGC the entanglement between the atom and its spontaneous emission fields is completely phase dependent, while in absence of this coherence the phase dependence of the entanglement disappears. Moreover, the degree of entanglement dramatically changes by the coherent superposition of the atomic states.

Keywords: Quantum entanglement; Quantum interference; Quantum entropy

Introduction

It is well known that the photon can be created when the atom decays from an upper level to a lower one [1-3]. It is also believed that the spontaneous emission destroys coherence in an atomic system, but the spontaneous emission has potential application in many physics processes; such as high-precision measurement, lasing without inversion, quantum teleportation, quantum computation, and quantum information theory [4]. Spontaneous emission, however, is used for produce atomic coherence as long as there exist two close-lying levels with non-orthogonal dipoles in an atomic system. Atomic coherence based on the spontaneous emission is usually referred to as vacuum-induced coherence or spontaneously generated coherence (SGC) [5]. Therefore, spontaneous decay can produce quantum interference. This happens in two cases, one occurs when an excited state doublet decays to a single ground state [6,7], and the other appears when a single exited state decays to a lower state doublet [5]. The effect of SGC can substantially modify the behavior of the system. In recent years, spontaneous emission has widely been used for various purposes [8, 9]. For example, the dissipation from the spontaneous emission can induce a transient entanglement between the two atoms, which is essential to implementation of quantum protocol such as quantum computation [10]. Moreover, entangled light can be created by the dissipation from white noise of the spontaneous emission [8]. The effect of quantum interference on the entanglement of a pair of three-level atoms has been proposed [11]. Quantum entangled states play an important role in the field of quantum information theory; particularly, quantum teleportation, quantum computation, etc [12,13]. In such a state, the system is inseparable and each component

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does not have properties independent of the other component. Phoenix and Knight [14,15] have shown that the reduced entropy is an accurate measure of entanglement between two components. Furthermore, the evolution of the atomic (field) entropy for the three-level atom (one– and two mode model) has been studied [16-20]. The time evolution of the field (atom) quantum entropy reflects time evolution of the degree of the entanglement of field (atom). The higher the entropy, the greater the entanglement.

In this article, the entanglement between the atom and its spontaneous emission fields is studied by means of quantum entropy. We show that in the presence of SGC, entanglement of atom and spontaneous emission strongly depends on relative phase of driving fields. In the absence of this coherence the phase dependence of the medium disappears. Moreover, the effect of atomic parameters such as Rabi- frequency and frequency detuning on quantum entropy are discussed.

Materials and Methods

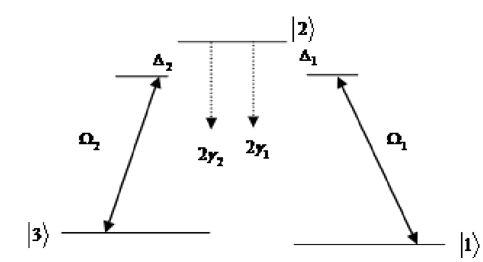
Consider a closed three-level Λ-type atomic system with two closely lower levels $|1\rangle$, $|3\rangle$ and the upper level $|2\rangle$ as shown in Figure 1(a). Two strong coherent coupling fields of frequencies ν_1 and ν_2 couple the $|1\rangle \to |2\rangle$ and $|3\rangle \to |2\rangle$ transitions, respectively. The corresponding Rabi-frequencies are denoted by $\Omega_1 = \frac{\vec{E}_1 \cdot \vec{\wp}_{12}}{2\hbar}$ and $\Omega_2 = \frac{\vec{E}_2 \cdot \vec{\wp}_{32}}{2\hbar}$. The parameters $\vec{\wp}_{j2}$ ($j = 1, 3$) denote the atomic dipole moments, but E_1 and E_2 represent the amplitudes of the coupling fields.

The density matrix equations of motion in the rotating wave approximation and in the rotating frame are [21, 22].

$$\begin{aligned}
\dot{\rho}_{11} &= 2\gamma_1 \rho_{22} - i\,\Omega_1 (\rho_{12} - \rho_{21}), \\
\dot{\rho}_{22} &= -2(\gamma_1 + \gamma_2)\rho_{22} + i\,\Omega_1(\rho_{12} - \rho_{21}) + i\,\Omega_2(\rho_{32} - \rho_{23}), \\
\dot{\rho}_{33} &= 2\gamma_2 \rho_{22} - i\,\Omega_2(\rho_{32} - \rho_{23}), \\
\dot{\rho}_{21} &= -(\gamma_1 + \gamma_2 - i\,\Delta_1)\rho_{21} + i\,\Omega_2 \rho_{31} + i\,\Omega_1(\rho_{11} - \rho_{22}), \\
\dot{\rho}_{13} &= -i\,(\Delta_1 - \Delta_2)\rho_{13} - i\,\Omega_2 \rho_{12} + i\,\Omega_1 \rho_{23} + 2\eta\sqrt{\gamma_1\gamma_2}\,\rho_{22}, \\
\dot{\rho}_{23} &= -(\gamma_1 + \gamma_2 - i\,\Delta_2)\rho_{23} + i\,\Omega_1 \rho_{13} - i\,\Omega_2(\rho_{22} - \rho_{33}).
\end{aligned} \qquad (1)$$

Here $\gamma_1 = \frac{\wp_{21}\,\omega_{21}^3}{3\pi\varepsilon_0 c^3}$ ($\gamma_2 = \frac{\wp_{23}\,\omega_{23}^3}{3\pi\varepsilon_0 c^3}$) are the spontaneous decay rates in transition $|2\rangle \to |j\rangle$ ($j = 1,3$). The detuning parameters are defined as $\Delta_1 = \nu_1 - \omega_{21}$ and $\Delta_2 = \nu_2 - \omega_{23}$. The term $(2\eta\sqrt{\gamma_1\gamma_2}\,\rho_{22})$ represents the interference among decay channels that appears due to SGC. The parameter $\eta (= \frac{\vec{\wp}_{23} \cdot \vec{\wp}_{21}}{|\vec{\wp}_{23}||\vec{\wp}_{21}|} = \cos\theta)$ denotes the alignment of the two dipole moments $\vec{\wp}_{23}$ and $\vec{\wp}_{21}$, where θ is the angle between two induced dipole moments $\vec{\wp}_{23}$ and $\vec{\wp}_{21}$ as shown in Fig. 1(b). Since the existence of SGC effect depends on the non-orthogonality of the two dipole moments $\vec{\wp}_{23}$, and $\vec{\wp}_{21}$, so we have to consider an arrangement where each field acts only on one transition (Fig.1 (b)). Moreover, η represents the strengths of the interference in spontaneous emission. For parallel dipole moments the interference is maximum and $\eta = 1$, while for perpendicular dipole moments there is no interference and $\eta = 0$. Therefore, η represents the existence of the SGC, and it will be zero (one) if the SGC effect is ignore (included). We note that only for nearly degenerate lower levels, i.e. $\omega_{21} \approx \omega_{23}$, the effect of SGC becomes important and for large lower energy levels separation it may be dropped [21, 23]. In the Λ-type atomic system considered here, an extra coherence term (SGC) appears between the lower levels due to the spontaneous decay from the upper level. The Rabi

(a)



(b)

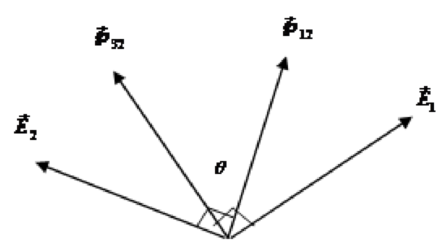


Figure 1. (a) Proposed level scheme. A Λ-type three-level atomic system driven by coherent fields. (b) The arrangement of field polarization required for a single field driving one transition if dipoles are orthogonal.

frequencies are connected to parameter η by the relation $\Omega_1 = g_1\sqrt{1-\eta^2}$ and $\Omega_2 = g_2\sqrt{1-\eta^2}$. Note that the phase appears in the equation through η. If we use $g_1 = |g_1|e^{-i\varphi_1}$, $g_2 = |g_2|e^{-i\varphi_2}$ and redefining the atomic variable in equation (1) as $\rho_{32} = \tilde{\rho}_{32}e^{-i\varphi_2}$, $\rho_{12} = \tilde{\rho}_{12}e^{-i\varphi_1}$, $\rho_{31} = \tilde{\rho}_{31}e^{-i\Delta\varphi}$, we obtain equations for the redefined density matrix elements ρ_{ij}. The equations are identical to equations (1) expect that η is replaced by

$$\eta \rightarrow \eta e^{-i\Delta\varphi}, \tag{2}$$

where $\Delta\varphi = \varphi_1 - \varphi_2$. We assume that the Λ-type three-level atom and the radiation-field reservoir are initially in a non-entangled pure state. So, the system is a bicomponent quantum system in a pure state. For such a system, the reduced quantum entropy can be used as a measure of the degree of entanglement between an atom and its spontaneous emission fields [14, 15]. The reduced entropy of the atom, i.e. $S_a(t)$, can be defined through its respective reduced-density operator by

$$S_a(t) = -Tr(\rho_a \ln(\rho_a)). \tag{3}$$

Here, ρ_a is the reduced density operator of the atom with the elements given in equation (1) in which the Boltzman constant is set equals one. We can express the Λ-type three-level atomic quantum entropy in terms of the eigenvalues $\lambda_a(t)$ of reduced atomic density operator ρ_a as

$$S_a(t) = -\sum_{i=1}^{3}\lambda_a(t)\ln(\lambda_a(t)).$$

Results and Discusion

In this section, we numerically calculate the entanglement between the atom and it spontaneous emission fields via equations (1) and (3). The influence of quantum interference due to SGC on entanglement of the atom and spontaneous emission fields is then discussed. We display the quantum entropy of the Λ-type three-level atom versus the normalized time γt in Fig. 2(a-c) for different atomic initial states, various g_1 and g_2, with $\eta = 0$ (or 0.99). Fig. 2(a) shows that for $|g_1| = |g_2| = 0$, and $\eta = 0$ the atomic quantum entropy quickly rises from zero to its maximum and reaches to a fixed value as time increases. So, the Λ-type three-level atom and its spontaneous emission fields are strongly entangled at the steady state. The degree of entanglement depends on the initially superposition of the atomic states. For atom initially in upper level $|2\rangle$, the degree of entanglement is larger than the case the atom is initially prepared in superposition states. In fact, the coherent superposition of the atomic states leads to

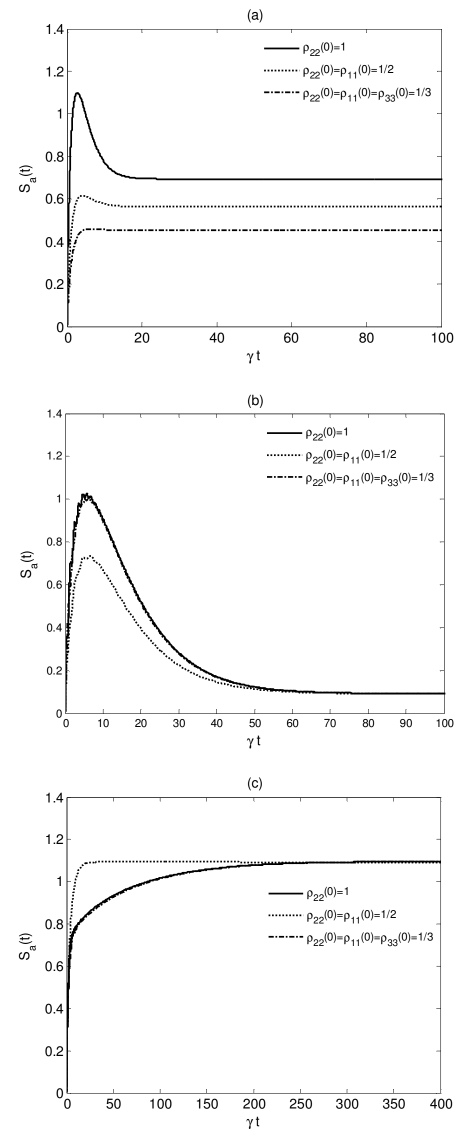


Figure 2. The time evolution of the atomic quantum entropy as a function of normalized time γt, (a) $|g_1| = |g_2| = 0$, $\eta = 0$ (b) $|g_1| = |g_2| = 2\gamma$, $\eta = 0$ (c) $|g_1| = |g_2| = 2\gamma$, $\eta = 0.99$. The other parameters are $\gamma_1 = 0.1\gamma$, $\gamma_2 = 0.1\gamma$, $\Delta_1 = 0$, $\Delta_2 = 0.5\gamma$ and $\Delta\varphi = 0$.

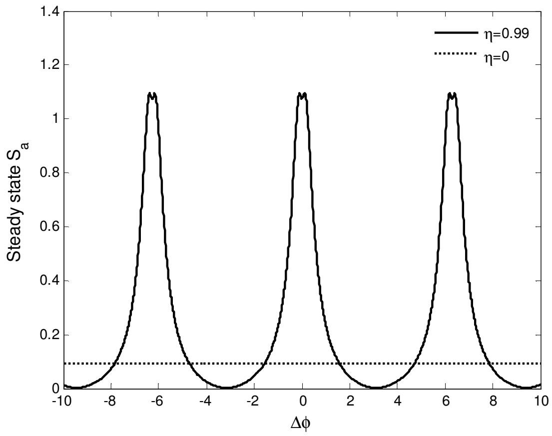


Figure 3. The steady state atomic quantum entropy as a function of $\Delta\varphi$ for $\eta = 0.99$ (solid line) and $\eta = 0$ (dotted line). The other parameters are same as Fig. 2.

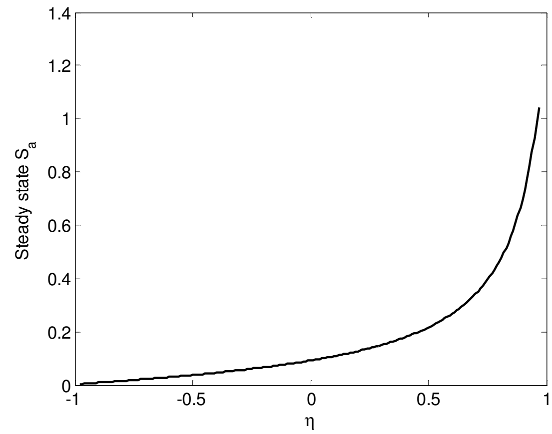


Figure 4. The steady state atomic quantum entropy as a function of as a function of η for $\Delta\varphi = 0$. The other parameters are same as Fig. 2.

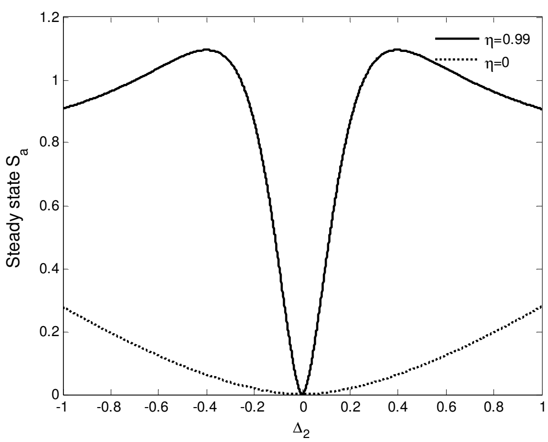


Figure 5. The steady state atomic quantum entropy as a function of as a function of Δ_2 for $\eta = 0.99$ (solid line), and $\eta = 0$ (dotted line). The other parameters are same as Fig. 2.

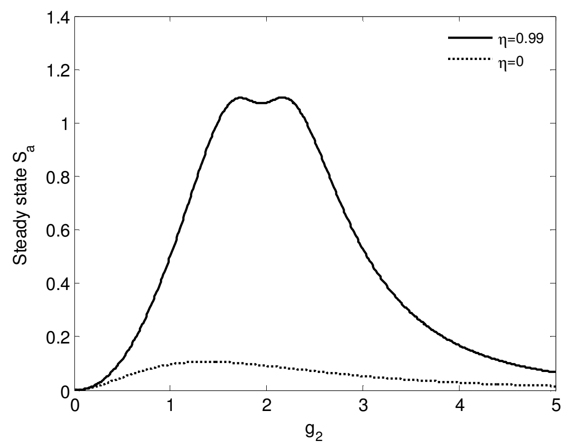


Figure 6. The steady state atomic quantum entropy as a function of as a function of g_2 for $\eta = 0.99$ (solid line) and $\eta = 0$ (dotted line). The other parameters are same as Fig. 2.

decrease the population of the atomic excited state and consequently reduction the probability of atomic spontaneous emission. This may lead in reduction of entanglement between the atom and its spontaneous emission. Fig. 2(b, c) shows that in the presence of coupling fields, i.e. $|g_1| = |g_2| = 2\gamma$, and $\eta = 0$ (or 0.99), the steady state entanglement does not depend on the initially preparation of atom. However, for $\eta = 0.99$ the degree of entanglement is larger than $\eta = 0$ (see Fig. 2(b, c)).

Now, we propose the effect of the atomic parameters on the entanglement between the atom and spontaneous emission fields. It has already been shown that the Λ-type three-level atomic system with SGC is phase dependent [23], and phase appears in the equations through η. So, the entanglement between the atom and its spontaneous emission fields should depend on the relative phase between applied fields. The phase variation of the entanglement for different values of quantum interference parameter is shown in Figure 3. Note that in the absence of quantum interference, i.e.

$\eta = 0$, the entanglement of the atom and the fields is phase independent (dotted line), while for $\eta = 0.99$, the entanglement substantially changes by the change of relative phase of applied fields (solid line). We realized that for even multiples of π, the atom and fields are strongly entangled, while for odd multiples of π, atom is disentangled from the spontaneous emission fields. Physically, the change of phase difference between applied fields may change the direction of the dipole moments: thus it changes parameter η.

Note that the degree of entanglement strongly depends on strength of the quantum interference η.

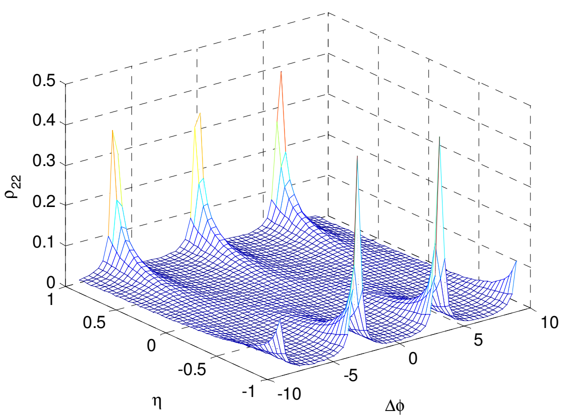


Figure 7. The steady state population distribution of upper level as a function of η and $\Delta\varphi$. The other parameters are same as Fig. 2.

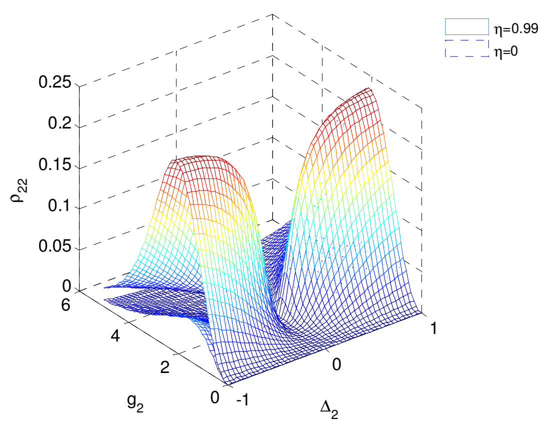


Figure 8. The steady state population distribution of upper level as a function of Δ_2 and g_2. The other parameters are same as Fig. 2.

Figure 4 shows that for $\Delta\varphi = 0$ the degree of entanglement increases by increasing η. Frequency detuning has an important role in creation of the entanglement between atom and its spontaneous emission fields. We show the steady state entropy $S_a(t)$ as a function of Δ_2 in the Figure 5. It is seen that the atom and its spontaneous emission fields are disentangled for $\Delta_2 = 0$. But, around zero detuning the degree of entanglement for $\eta = 0.99$ is larger than $\eta = 0$.

The degree of entanglement can also be changed by the Rabi-frequencies of applied fields. The quantum entropy versus g_2 is displayed in Figure 6. It can also be realized that for $\eta = 0.99$ the degree of entanglement is larger than $\eta = 0$. A similar behavior can be found for entanglement of the atom and its spontaneous emission fields by variation g_1.

Now, we discuss the physical mechanisms underlying the behind of the above results. Here, we plot the population distribution of level $|2\rangle$ versus various atomic parameters. Figure 7 shows the two dimensional behavior of the population distribution of level $|2\rangle$ versus η and $\Delta\varphi$. We observe that the peaks of upper level population are located in places that the quantum entropies are maxima (see Figs. 3, 4 and Fig. 7). The higher the population, the greater the entanglement. Physically, increasing the population of level $|2\rangle$ can increase the probability of atomic spontaneous emission leading to increase in entanglement. Finally, the variation of upper level population versus g_2 and Δ_2 is shown in Figure 8. For $\Delta_2 = 0$ the population of upper level is approximately zero, and the spontaneous emission is suppressed. In this case the atom is disentangled from the spontaneous emission fields. However, for $\Delta_2 \neq 0$ the upper level $|2\rangle$ is populated leading to creation an entanglement between the atom and fields (see Figs. 5, 6, and 8).

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