LASERS WITHOUT INVERSION: DENSITY OPERATOR METHOD

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Abstract

A quantum theory of a two and three-level laser with injected atomic coherence is developed by using a density operator method, to the best of our knowledge, for the first time. The initial atomic coherence plays an essential role. At steady state, the equation of motion for the density operator yields to exhibit laser without inversion and a phase locking but no threshold for the laser field. The Fokker-Planck equation is also derived for three level lasers and the laser action is analyzed in terms of the coefficients of this equation which results in the quantum noise quenching. One of the most important aspects of our method is its capability to analyze the coherently pumped laser action in N-level systems in a simple fashion.

Introduction

The semiclassical and quantum theories of the laser were developed more than 20 years ago [1-3]. Some of the key concepts of the theoretical description are population inversion and laser threshold.

It is generally the case that a laser requires population inversion in order to overcome the absorption from the lower level, since the gain is proportional to the population difference between the upper and lower levels of the lasing transition. Additionally, in an ordinary laser, where atoms are incoherently pumped to their upper levels, there are amplitude and phase fluctuations, due to the spontaneous emission, which in turn lead to uncertainty in the photon number distribution and linewidth of the laser, respectively.

During the last few years it has been proposed that laser action can be achieved even if the usual population inversion between the lasing levels does not occur [4-11]. The essential point for this possibility is to modify the emissive and absorptive profiles, with the help of a quantum interference effect. In these laser systems, if atomic coherence, a kind of quantum interference, is produced in lower levels, it leads to an absorption cancellation. A small population in the excited state can thus lead to net gain. According to the crucial role of the atomic coherence in the laser process, a quantum theory of a two-level single mode laser with injected atomic coherence has been developed by generalizing the Scully-Lamb laser theory and it has been shown that the injected atomic coherence reduces both the photon-number noise and phase-noise simultaneously [9].

In this paper, a quantum theory of a two and three-level laser with injected atomic coherence is developed by using the density operator method for the first time. We derive the density operator equation of motion for pumping field. At steady state, this equation leads to lasing without inversion, phase locking and elimination of the threshold of the field. Also, in the case of the three-level laser with injected atomic coherence, we will show quantum noise quenching. Our method exhibits the role of the atomic coherence explicity, and more importantly, it can be generalized for N-level systems.

Keywords: Density operator; Lasers without inversion; Noise quenching; Phase locking

Density Operator Formalism

In Heisenberg picture, the density operator for continuous states $|\zeta\rangle$ with distribution function $P(\zeta, t)$ is given by [1]:

$$\rho(t)=\int d^2\zeta P(\zeta,t)|\zeta\rangle\langle\zeta| \quad (1)$$

and in interaction picture is given by [1]:

$$\rho(t)=\int d^2\zeta P(\zeta)|\zeta(t)\rangle\langle\zeta(t)| \quad (2)$$

where P (ζ) is given by:

$$P(\zeta)=\frac{1}{1+N_0}\exp\left(\frac{-\zeta\zeta^*}{1+N_0}\right) \quad (3)$$

N_0 is normalized photons number in steady state.

In general, we assume that system A, with density operator $\rho_A(t)$, interacts with system B, with density operator $\rho_B(t)$. The density operator for the combined system A-B is given by an outer product of $\rho_A(t)$ and $\rho_B(t)$ [1]:

$$\rho_{A\text{-}B}(t)=\rho_A(t)\otimes\rho_B(t) \quad (4)$$

The density operator for system A (or B) is then given by the trace over B (or A) coordinates

$$\rho_A(t)=Tr_B\{\rho_{A\text{-}B}(t)\}=\Sigma_B\langle B|\rho_{A\text{-}B}(t)|B\rangle \quad (5)$$

After a short time τ from initial time t, the density operator of system A-B can be expanded about its initial value:

$$\rho_{A\text{-}B}(t+\tau)=\rho_{A\text{-}B}(t)+\dot{\rho}_{A\text{-}B}(t)|_t^{t+\tau}+\ddot{\rho}_{A\text{-}B}(t)|_t^{t+\tau}+\ldots \quad (6)$$

The Heisenberg equation of motion for ρ (t) is given by [3]:

$$\dot{\rho}_{A\text{-}B}(t)=\frac{-i}{\hbar}[H_{A\text{-}B},\rho_{A\text{-}B}] \quad (7)$$

where $H_{A\text{-}B}$ is the Hamiltonian of system A and system B. Using (6) and (7), we obtain:

$$\rho_A(t+\tau)=Tr_B[\rho_{A\text{-}B}(t)] +\Sigma_{(k=1)}\left(\frac{-i}{\hbar}\right)^k\int_t^{t+\tau}dt_1\ldots\int_t^{t+\tau}dt_k \quad (8)$$
$$Tr_B\{[H_{A\text{-}B}(t_1),\ldots,[H_{A\text{-}B}(t_k),\rho_{A\text{-}B}(t)]\ldots]\}$$

using (5) and assuming that the time variation of $H_{A\text{-}B}$ is small during time τ, we have:

$$\rho_A(t+\tau)=\rho_A(t)+ \Sigma_{(k=1)}\left(\frac{-i}{\hbar}\right)^k\int_t^{t+\tau}dt_1\ldots\int_t^{t+\tau}dt_k \quad (9)$$
$$Tr_B\{[H_{A\text{-}B}(t_1),\ldots,[H_{A\text{-}B}(t_k),\rho_{A\text{-}B}(t)]\ldots]\}$$

Now, we denote A as the field and B as an atomic system.

In a period of τ, and because of the atom field interaction, r_a atoms are pumped to upper level $|a\rangle$. Thus we have:

$$\dot{\rho}_A(t)=r_a[\rho_A(t+\tau)-\rho_A(t)] \quad (10)$$

combining (9) and (10) and ignoring the powers greater than two in expansion, we have:

$$\dot{\rho}_A(t)=\left(\frac{-i}{\hbar}\right)r_a\tau Tr_B\{[H_{A\text{-}B},\rho_{A\text{-}B}(t)]\} - \left(\frac{-1}{\hbar^2}\right)r_a\tau^2 Tr_B\{[H_{A\text{-}B},[H_{A\text{-}B},\rho_{A\text{-}B}(t)]]\} \quad (11)$$

Equation (11) is the density operator equation for pumping field.

Coherently Pumped Two-Level Laser

In this section, we consider a two-level atomic beam with upper level $|a\rangle$ and lower $|b\rangle$ which is pumped coherently. Then we inject this atomic beam into a cavity as shown in Figure 1.

The density matrix of coherent atomic beam then is given by:

$$\rho_{atom}=\begin{bmatrix}\rho_{aa} & \rho_{ab}\\ \rho_{ba} & \rho_{bb}\end{bmatrix} \quad (12)$$

where ρ_{ab} and ρ_{ba} are coherence terms.

In laser cavity the atomic beam interacts with field. Thus, the interaction Hamilton is given by [3]:

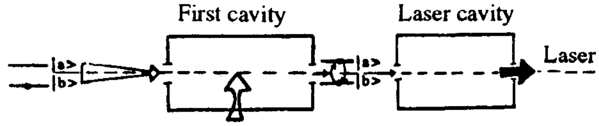

Figure 1. Scheme of the coherently pumped laser. The atoms in the first cavity are prepared coherently and then injected into the laser cavity.

$$H_{a\text{-}f} = \hbar g\,(\sigma a^\dagger + \sigma^\dagger a) \quad (13)$$

Using (4), the atom-field density operator is given by:

$$\rho_{a\text{-}f}(t) = \rho_{atom} \otimes \rho(t) = \begin{bmatrix} \rho_{aa}\,\rho(t) & \rho_{ab}\,\rho(t) \\ \rho_{ba}\,\rho(t) & \rho_{bb}\,\rho(t) \end{bmatrix} \quad (14)$$

Applying (13) and (14) in (11) and adding cavity loss term we have:

$$\dot\rho(t) = \{-is\rho_{ab}[a,\rho] - \frac{\alpha}{2}[\rho_{aa}(aa^\dagger\rho - a^\dagger\rho a) + \rho_{bb}(a^\dagger a\rho - a\rho a^\dagger)]\} - \frac{\gamma}{2}\rho(t)a^\dagger a + c.c \quad (15)$$

where $s = r_a g\tau = r_a \frac{g}{\Gamma}$ represents the strength of the deriving field, $\alpha = 2 r_a g^2 \tau^2 = 2 r_a \frac{g^2}{\Gamma^2}$ is the linear gain coefficient, γ is loss constant of cavity and Γ is the decay rate of atomic levels.

The eigenstate $|\zeta\rangle$ can be expanded on the base ket of simple harmonic oscillator [3]. So, we have:

$$|\zeta\rangle = \exp[\frac{-1}{2}\zeta\zeta^*]\sum_{n=0}\frac{(\zeta a^\dagger)^n}{n!}|0\rangle = \exp[-\frac{1}{2}\zeta\zeta^*]\exp[\zeta a^\dagger]|0\rangle \quad (16)$$

where a ($a^\dagger$) is field annihilation (creation) operator and ζ is eigenvalue of a with eigenstate $|\zeta\rangle$.

From (16) we obtain:

$$a^\dagger(|\zeta\rangle\langle\zeta|) = (\frac{\partial}{\partial\zeta} + \zeta^*)|\zeta\rangle\langle\zeta| \quad (17)$$

$$|\zeta\rangle\langle\zeta|a = (\frac{\partial}{\partial\zeta^*} + \zeta)|\zeta\rangle\langle\zeta| \quad (18)$$

and

$$\frac{\partial}{\partial t}(|\zeta\rangle\langle\zeta|) = |\zeta\rangle\langle\zeta|a^\dagger\frac{da}{dt} + \frac{da^\dagger}{dt}a|\zeta\rangle\langle\zeta| = |\zeta\rangle\langle\zeta|a^\dagger\frac{da}{dt} + c.c \quad (19)$$

Using (2), we obtain the following relation in interaction picture:

$$\dot\rho(t) = \int d^2\zeta P(\zeta)\frac{\partial}{\partial t}[|\zeta(t)\rangle\langle\zeta(t)|] \quad (20)$$

Applying (19) in (20) we have:

$$\dot\rho(t) = \int d^2\zeta P(\zeta)[|\zeta(t)\rangle\langle\zeta(t)|a^\dagger\frac{da}{dt}] + c.c \quad (21)$$

And using (2) and (21) in (15) we obtain:

$$\int d^2\zeta P(\zeta)[|\zeta(t)\rangle\langle\zeta(t)|a^\dagger\frac{da}{dt}] = \int d^2\zeta\{-is\rho_{ba}P(\zeta)(a|\zeta\rangle\langle\zeta| - |\zeta\rangle\langle\zeta|a) - \frac{\alpha}{2}\rho_{aa}P(\zeta)(aa^\dagger|\zeta\rangle\langle\zeta| - a^\dagger|\zeta\rangle\langle\zeta|a) + \rho_{bb}P(\zeta)(a^\dagger a|\zeta\rangle\langle\zeta| - a|\zeta\rangle\langle\zeta|a^\dagger)]\} - \frac{\gamma}{2}P(\zeta)[|\zeta(t)\rangle\langle\zeta(t)|a^\dagger a] \quad (22)$$

Then applying (17) and (18) in (22) we have:

$$\dot\rho(t) = \int d^2\zeta P(\zeta)|\zeta(t)\rangle\langle\zeta(t)|a^\dagger\frac{da}{dt} = \int d^2\zeta\{-is\rho_{ab}P(\zeta)\frac{\partial}{\partial\zeta^*}[|\zeta(t)\rangle\langle\zeta(t)|] + \frac{\alpha}{2}(\rho_{aa} - \rho_{bb})P(\zeta)\zeta\frac{\partial}{\partial\zeta}[|\zeta(t)\rangle\langle\zeta(t)|] + \frac{\alpha}{2}\rho_{aa}P(\zeta)\frac{\partial^2}{\partial\zeta\partial\zeta^*}[|\zeta(t)\rangle\langle\zeta(t)|]\} - \frac{\gamma}{2}P(\zeta)(\zeta\frac{\partial}{\partial\zeta} + \zeta^*\zeta)|\zeta(t)\rangle\langle\zeta(t)|] \quad (23)$$

Also, we know that the distribution function $P(\zeta)$ is zero at infinity [3]:

$$\int d^2\zeta\frac{\partial}{\partial\zeta}[P(\zeta)|\zeta\rangle\langle\zeta|] = [P(\zeta)|\zeta\rangle\langle\zeta|]_{-\infty}^{+\infty} = 0 \quad (24)$$

therefore, (23) becomes:

$$\dot{\rho}(t)=\int d^2\zeta P(\zeta)|\zeta(t)\rangle\langle\zeta(t)|a^\dagger\frac{da}{dt}$$

$$=\int d^2\zeta\{(-\frac{\partial}{\partial\zeta})[\frac{\alpha}{2}(\rho_{aa}-\rho_{bb})\zeta-is\,\rho_{ab}]P(\zeta) \tag{25}$$

$$+\frac{\alpha}{2}\rho_{aa}\frac{\partial^2P(\zeta)}{\partial\zeta\partial\zeta^*}+\frac{\gamma}{2}\frac{\partial}{\partial\zeta}[\zeta P(\zeta)]$$

$$-\frac{\gamma}{2}\zeta^*\zeta P(\zeta)\}\,|\zeta(t)\rangle\langle\zeta(t)|$$

The eigenvalue of **a** (i.e. ζ) can be written as:

$$\zeta=\sqrt{n_0}\,e^{i\phi_0} \tag{26}$$

where n_0 and ϕ_0 are average photon number and field phase in steady state, respectively.

In steady state (i.e. da/dt= 0) and using (2) and (26) and assuming classical field (n_0>>1), relation (25) can be written as:

$$\frac{\alpha}{2}(\rho_{aa}-\rho_{bb})\frac{n_0}{1+N_0}-is\rho_{ab}\frac{\sqrt{n_0}\,e^{-i\phi_0}}{1+N_0}-\frac{\gamma}{2}n_0$$
$$+\frac{n_0}{2}[\frac{\alpha\rho_{aa}}{(1+N_0)^2}-\frac{\gamma}{(1+N_0)}]=0 \tag{27}$$

on the other hand α, γ, ρ_{aa} and N_0 are in the same order of magnitude:

$$0\le\alpha,\gamma,\rho_{aa},N_0\le 1 \tag{28}$$

So we have:

$$\frac{\alpha\rho_{aa}}{(1+N_0)^2}-\frac{\gamma}{(1+N_0)}\simeq 0 \tag{29}$$

This relation is in agreement with the fact that by increasing γ, for a fixed value of α, ρ_{aa} is increased (increasing ρ_{aa} corresponds to increasing absorption).

From (29) and (27) we get:

$$\frac{\alpha}{2}\frac{(\rho_{aa}-\rho_{bb})\sqrt{N_0}-2i\alpha|\rho_{ab}|\,e^{-i(\theta-\phi_0)}}{1+N_0}-\frac{\gamma}{2}\sqrt{N_0}=0 \tag{30}$$

where we made use of the following relation:

$$\rho_{ab}=|\rho_{ab}|e^{i\theta} \tag{31}$$

$$n_0=(\frac{s}{\alpha})^2N_0 \tag{32}$$

Relation (30) is valid whenever the real and imaginary parts are zero separately.
Thus, we have:

$$\cos(\theta-\phi_0)=0 \quad\Rightarrow\quad \theta-\phi_0=\pm\frac{\pi}{2} \tag{33}$$

with the negative sign, the stability of field is not satisfied (if $\phi_0\Rightarrow\phi_0+\delta\phi$ in field phase). So:

$$\phi_0=\theta-\frac{\pi}{2} \tag{34}$$

This shows the phase locking in coherently pumped two-level laser. By putting the real part of (30) equal to zero and using (34), we obtain:

$$\frac{(\rho_{aa}-\rho_{bb})\sqrt{N_0}+2|\rho_{ab}|}{1+N_0}=\frac{\gamma}{\alpha}\sqrt{N_0} \tag{35}$$

This relation shows the laser without inversion. In other words, even if $\rho_{aa}<\rho_{bb}$, we still have a positive value for N_0. This is shown in Figure 2. From (35), we can also show there is no threshold for pumping field. This is shown in Figure 3. Relation (35) was derived by Ning Lu using a different method [9].

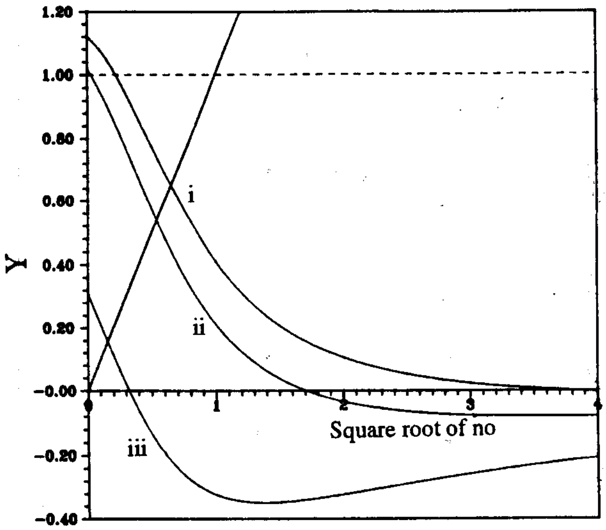

Figure 2. The left and right-hand sides of equation (35) as a function of a normalized laser amplitude $\sqrt{N_0}$, γ/α= 1 and left-hand side curves are plotted for i, ρ_{aa} = .03 ii, ρ_{aa} = 0.1 iii, ρ_{aa} = 0.01.

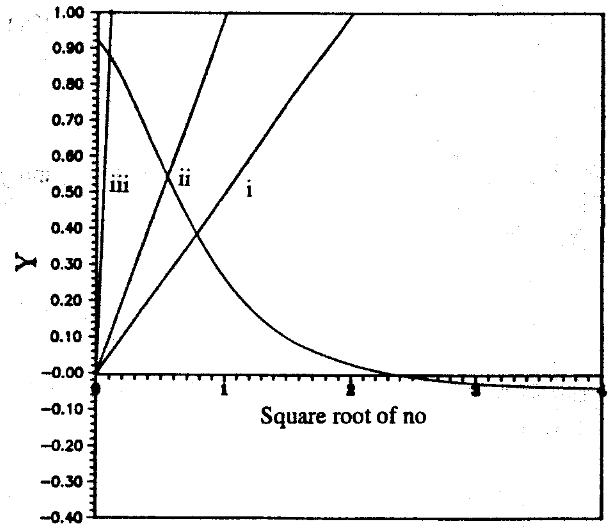

Figure 3. The left and right-hand sides of equation (35) as a function of a normalized laser amplitude $\sqrt{N_0}$ for fixed $\rho_{aa} = 0.3$, but for various values of γ/α: i, $\gamma/\alpha = 0.5$ ii, $\gamma/\alpha = 1$ iii, $\gamma/\alpha = 10$.

Coherently Pumped Three-Level Laser

In this section, we assume that the atomic beam consists of three-level, atoms (upper level $|a\rangle$, middle level $|b\rangle$ and lower level $|c\rangle$) which are prepared coherently and then are injected into a laser cavity (as shown in Fig. 1).

By using (4) the density operator for atom-field is given by:

$$\rho_{a-f} = \begin{bmatrix} \rho_{aa}\rho(t) & \rho_{ab}\rho(t) & \rho_{ac}\rho(t) \\ \rho_{ba}\rho(t) & \rho_{bb}\rho(t) & \rho_{bc}\rho(t) \\ \rho_{ca}\rho(t) & \rho_{cb}\rho(t) & \rho_{cc}\rho(t) \end{bmatrix} \tag{36}$$

Using the interaction Hamiltonian for atom-field, (36) and (11), we obtain:

$$\dot{\rho}(t) = \{ -is(\rho_{ba} + \rho_{cb})[a, \rho]$$

$$-\frac{\alpha}{2}[(\rho_{aa} + \rho_{bb})(aa^{\dagger}\rho - a^{\dagger}\rho a) + (\rho_{bb} + \rho_{cc})(\rho a^{\dagger}a - a\rho a^{\dagger})$$

$$+\rho_{ca}(aa\rho - a\rho a + \rho aa - a\rho a)]\} - \frac{\gamma}{2}\rho(t)a^{\dagger}a + c.c \tag{37}$$

Using the same argument as in the previous section and applying (17), (18), (21), (24), (26) in steady state (i.e. $da/dt = 0$), we obtain:

$$\frac{\alpha}{2}\frac{(\rho_{aa} - \rho_{cc})\sqrt{N_0} + 2[|\rho_{ab}| + |\rho_{bc}|]}{1 + N_0} - \frac{\gamma}{2}\sqrt{N_0}$$

$$+\frac{n_0}{2}\left[\frac{\alpha(\rho_{aa} + \rho_{bb})}{(1 + N_0)^2} - \frac{\gamma}{(1 + N_0)} - \frac{\alpha\rho_{ac}e^{-i2\phi_0}}{(1 + N_0)^2}\right] = 0 \tag{38}$$

where we have also used (32) and the following relations:

$$\rho_{ab} = |\rho_{ab}|e^{i\theta_{ab}} \tag{39}$$

$$\rho_{bc} = |\rho_{bc}|e^{i\theta_{bc}} \tag{40}$$

The bracket in (38) is almost zero because α, γ, ρ_{aa}, N_0 and $|\rho_{ac}|$ are in the same order of magnitudes. By putting the imaginary part of (38) to zero and considering the stability of field, we have:

$$\theta - \phi_0 = \theta_{ab} - \phi_0 = \theta_{bc} - \phi_0 = \frac{1}{2}\theta_{ac} - \phi_0 = \frac{\pi}{2} \tag{41}$$

This shows the phase locking in the coherently pumped three-level laser. By putting the real part of (38) equal to zero and using (41) we get:

$$\frac{\alpha}{2}\frac{(\rho_{aa} - \rho_{cc})\sqrt{N_0} + 2[|\rho_{ab}| + |\rho_{bc}|]}{1 + N_0} - \frac{\gamma}{2}\sqrt{N_0} = 0 \tag{42}$$

This shows there is lasing without inversion in the coherently pumped three-level laser which is also shown in Figure 4. Also (42) shows there is no threshold for pumping field. This is shown in Figure 5.

Quantum Noise Quenching in Coherently Pumped Three-Level Laser

Using Heisenberg picture, (1) and (37), we get:

$$\int d^2\zeta P(\zeta, t)|\zeta\rangle\langle\zeta| = \int d^2\zeta P(\zeta, t)\{-is(\rho_{ba} + \rho_{cb})[a, \rho]$$

$$-is(\rho_{ab} + \rho_{bc})[a^{\dagger}, \rho]$$

$$-\frac{\alpha}{2}[(\rho_{aa} + \rho_{bb})(\rho aa^{\dagger} - a^{\dagger}\rho a + \rho aa^{\dagger} - a^{\dagger}\rho a)$$

$$+(\rho_{bb} + \rho_{cc})(\rho a^{\dagger}a - a\rho a^{\dagger} + a^{\dagger}a\rho - a\rho a^{\dagger})$$

$$+\rho_{ca}(aa\rho - a\rho a + \rho aa - a\rho a)$$

$$+\rho_{ac}(a^{\dagger}a^{\dagger}\rho - a^{\dagger}\rho a^{\dagger} + \rho a^{\dagger}a^{\dagger} - a^{\dagger}\rho a^{\dagger}) \tag{43}$$

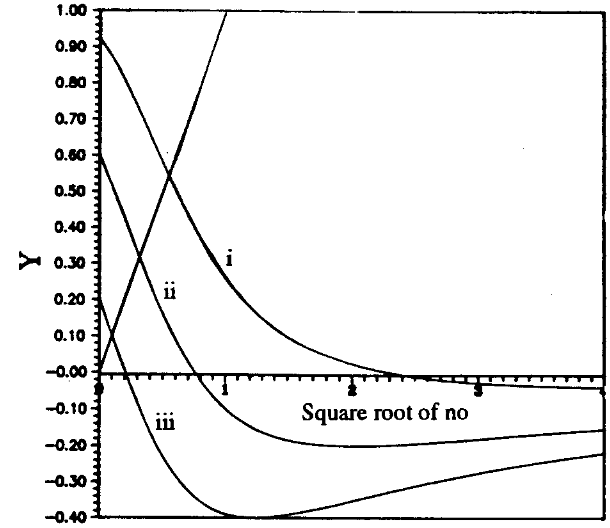

Figure 4. Equation (42) as a function of a normalized laser amplitude, $\sqrt{N_0}$ for fixed $\gamma/\alpha = 1$ but for various values of ρ_{aa} and ρ_{bb}: i $\rho_{bb}=0.2, \rho_{aa}=0.3$, ii $\rho_{bb}=0.1, \rho_{aa}=0.2$, iii $\rho_{bb}=0.01$, $\rho_{aa}=0.02$.

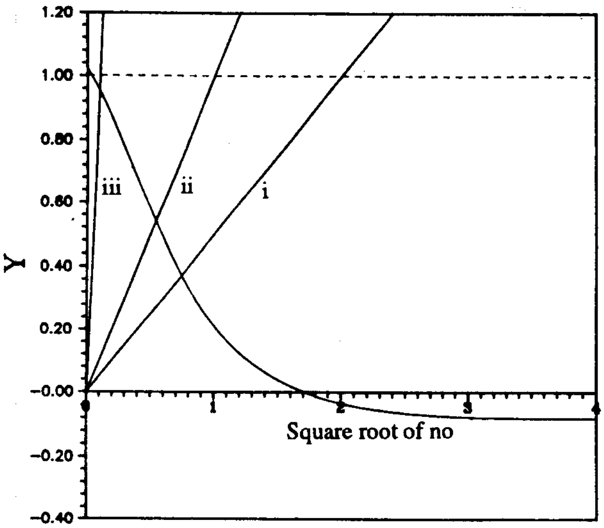

Figure 5. Equation (42) as a function of a normalized laser amplitude, $\sqrt{N_0}$ for $\rho_{bb}=0.2$, $\rho_{aa}=0.1$, but for various values of γ/α: i, $\gamma/\alpha = 0.5$ ii, $\gamma/\alpha = 1$ iii, $\gamma/\alpha = 10$.

$$-\frac{\gamma}{2}[|\zeta\rangle\langle\zeta|a^\dagger a + a^\dagger a|\zeta\rangle\langle\zeta|]\}$$

using (17), (18) and (24), we have:

$$\begin{aligned}P(\zeta,t) = & is(\rho_{ab}+\rho_{bc})\frac{\partial P(\zeta,t)}{\partial\zeta} - is(\rho_{ba}+\rho_{cb})\frac{\partial P(\zeta,t)}{\partial\zeta^*} \\ & -\frac{\alpha}{2}(\rho_{aa}-\rho_{cc}-\frac{\gamma}{\alpha})\{\frac{\partial}{\partial\zeta}[\zeta P(\zeta,t)] + \frac{\partial}{\partial\zeta^*}[\zeta^* P(\zeta,t)] \\ & +\alpha(\rho_{aa}+\rho_{bb})\frac{\partial^2 P(\zeta,t)}{\partial\zeta\partial\zeta^*} - \frac{\alpha}{2}\rho_{ac}\frac{\partial^2 P(\zeta,t)}{\partial\zeta^2} \\ & -\frac{\alpha}{2}\rho_{ca}\frac{\partial^2 P(\zeta,t)}{\partial\zeta^{*2}} + \gamma\zeta^*\zeta P(\zeta,t)\end{aligned} \quad (44)$$

If r and ϕ show magnitude and phase of field respectively, we have:

$$\zeta = r e^{i\phi} \quad (45)$$

So, we find:

$$\frac{\partial}{\partial\zeta} = \frac{e^{-i\phi}}{2}(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\phi}) \quad (46)$$

$$\frac{\partial}{\partial\zeta^*} = \frac{e^{i\phi}}{2}(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\phi}) \quad (47)$$

$$\frac{\partial^2}{\partial\zeta\partial\zeta^*} = \frac{1}{4}(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}) \quad (48)$$

$$\frac{\partial^2}{\partial\zeta^2} = \frac{e^{-i2\phi}}{4}[\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + i(\frac{1}{r^2}\frac{\partial}{\partial\phi} - \frac{2}{r}\frac{\partial^2}{\partial r\delta\phi})] \quad (49)$$

using (46), (47), (48) and (49) in (44) and applying steady state we get:

$$\begin{aligned}P(r,\phi,t) = & \{\frac{1}{4}[\alpha(\rho_{aa}+\rho_{bb}) + \alpha|\rho_{ac}|]\frac{\partial^2 P(r,\phi,t)}{\partial r^2} \\ & +\frac{1}{4r}[\alpha(\rho_{aa}+\rho_{bb}) - \alpha|\rho_{ac}| - 2\alpha r^2(\rho_{aa}-\rho_{cc}-\frac{\gamma}{\alpha}) \\ & -4rs(|\rho_{ab}|+|\rho_{bc}|)\frac{\partial P(r,\phi,t)}{\partial r} \\ & -[\alpha(\rho_{aa}-\rho_{cc}-\frac{\gamma}{\alpha}) - \gamma r^2]P(r,\phi,t) \\ & +\frac{1}{4r^2}[\alpha(\rho_{aa}+\rho_{bb}) - \alpha|\rho_{ac}|]\frac{\partial^2 P(r,\phi,t)}{\partial\phi^2}\}\end{aligned} \quad (50)$$

this shows the Fokker-Planck equation for a coherently pumped three-level laser. Thus, the phase diffusion coefficient is equal to [1].

$$d = \frac{\alpha}{4 r_0^2} (\rho_{aa} + \rho_{bb} - |\rho_{ac}|) \frac{\partial^2 P(\phi, t)}{\partial \phi^2}$$
$$= \frac{\alpha}{4 n_0} (\rho_{aa} + \rho_{bb} - |\rho_{ac}|) \frac{\partial^2 P(\phi, t)}{\partial \phi^2} \qquad (51)$$

If ($\rho_{aa} + \rho_{bb} = |\rho_{ac}|$), then the phase diffusion coefficient and therefore quantum noise is equal to zero. This is called "quantum noise quenching." On the other hand, if ($\rho_{aa} + \rho_{bb} < |\rho_{ac}|$) then phase diffusion coefficient becomes negative. In this case, as a result of atom-field interaction the noise of system is less than the vacuum noise.

Conclusion

In summary, we have studied the operation of coherently pumped two and three-level lasers by using a density operator method. We showed that in the presence of atomic coherence, laser without population inversion is possible. The initial atomic coherence leads to laser phase locking and furthermore produces driving force, so that there is no threshold in the coherently pumped laser. These results were shown for both two and three level lasers. We also derived the Fokker-Planck equation for the three-level laser. By using this equation, we showed quantum noise quenching for coherently pumped three-level laser. Our method can be generalized for N-level lasers. Furthermore, we showed that the approach developed here leads to the same results as those obtained by Ning Lu [9].

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