

THE FRANK KINETIC MODEL FOR SPONTANEOUS CHIRAL SYMMETRY BREAKING WITH VARIABLE CONCENTRATION OF THE ACHIRAL SUBSTRATE

I. Gutman

*Institute of Physical Chemistry, Attila Jozsef University,
P.O. Box 105 H-6701 Szeged, Hungary*

Abstract

This paper considers the possibility of spontaneous transition of a racemic mixture (composed of equal amounts of two enantiomeric species, say L and D) into a monochiral state (in which only one enantiomer, say L, is present). Transitions of this kind have been recently experimentally observed. A plausible approach to this phenomenon is a model put forward by Frank, based on simple laws of chemical kinetics. In order to be able to solve the underlying system of differential equations, Frank made a drastic assumption, namely that the amount of achiral substrate from which the enantiomers L and D are formed is time-independent. In this paper, it is demonstrated that such an assumption is not justified, but that the essential features of the Frank model are also maintained if the amount of the achiral substrate is allowed to be time-dependent.

Introduction

The possibility of spontaneous transition of a physico-chemical system from a symmetric or racemic initial state into a chiral terminal state (the so-called chiral symmetry breaking) has intrigued the scientific community for more than a century [1-3]. In a number of recent researches [4-10], it was established that such transitions may indeed occur under well-defined laboratory conditions.

In 1953, Charles Frank [11] demonstrated that chiral symmetry breaking could be a consequence of the basic law of classical chemical kinetics, namely the law of mass action. Frank designed a simple kinetic scheme by which an almost racemic mixture evolves into a completely monochiral terminal state. In the following four decades, the Frank model was considered as just a curiosity in (theoretical) chemical kinetics, without a sound experi-

mental basis. In the last few years, however, several experimental realizations of Frank-type processes were reported [7, 9, 10], justifying further theoretical studies in this direction [12-15]. The present paper offers some observations on the Frank model and shows that it is more general than originally anticipated.

The Frank Model

The model proposed by Frank [11] is based on the application of the law of mass action to the following elementary chemical reactions:



L and D are enantiomers capable of self-replication (reactions (α') and (α'')); A stands for an achiral substrate. The

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two enantiomers are supposed to eliminate each other from the system via the formation of an inactive product, e.g. of an insoluble complex (reaction (β)). Observe that all the three elementary steps in the Frank model are assumed to be irreversible.

Applying the law of mass action to the reactions (α) and (β) and denoting by $c_X = c_X(t)$ the (time-dependent) concentration of the substance X, X=L, D, A, we readily arrive at the system of differential equations

$$dc_L/dt = k_1 c_A c_L - k_2 c_L c_D \quad (1a)$$

$$dc_D/dt = k_1 c_A c_D - k_2 c_L c_D \quad (1b)$$

$$dc_A/dt = -k_1(c_L + c_D) c_A \quad (1c)$$

Here k_1 and k_2 are the rate constants of the reactions (α) and (β), respectively; the reactions (α') and (α'') are, of course, supposed to have equal rate constants. The initial conditions are $c_L = c_{L0}$ and $c_D = c_{D0}$ for $t = 0$. Without loss of generality, throughout this paper it will be assumed that $c_{L0} \geq c_{D0}$.

In order to be able to solve the differential equations (1a) and (1b), Frank made the assumption that the concentration c_A of the substrate A is time-independent, and completely ignored equation (1c). It must be noted that this assumption is not chemically sound: If, namely, c_A were time-independent, then it would be $dc_A/dt = 0$; from equation (1c) we see that this would imply either $c_A = 0$ or $c_L = c_D = 0$, both conditions being chemically unacceptable.

If, nevertheless, c_A is considered to be time-independent [11], then the functions $c_L(t)$ and $c_D(t)$ are calculated as follows [11]. First subtract (1b) from (1a) which results in

$$d(c_L - c_D)/dt = k_1 c_A (c_L - c_D)$$

Straightforward integration yields

$$c_L - c_D = (c_{L0} - c_{D0}) \exp(k_1 c_A t) \quad (2)$$

Dividing (1a) with (1b) one obtains

$$dc_L/dc_D = c_L(k_1 c_A - k_2 c_D) / [c_D(k_1 c_A - k_2 c_L)]$$

which makes possible the separation of the variables c_L and c_D :

$$(k_1 c_A - k_2 c_L) dc_L/c_L = (k_1 c_A - k_2 c_D) dc_D/c_D$$

and then by direct integration:

$$k_1 c_A \ln(c_L/c_{L0}) - k_2(c_L - c_{L0}) = k_1 c_A \ln(c_D/c_{D0}) - k_2(c_D - c_{D0}) \quad (3)$$

Combining (2) and (3) one easily finds explicit analytical expressions for the functions $c_L(t)$ and $c_D(t)$:

$$c_L = c_{L0}(c_{L0} - c_{D0}) (c_{L0} - c_{D0} F)^{-1} \exp(k_1 c_A t) \quad (4')$$

$$c_D = c_{D0} (c_{L0} - c_{D0}) F (c_{L0} - c_{D0} F)^{-1} \exp(k_1 c_A t) \quad (4'')$$

where

$$F = \exp\left[-(k_1 c_A / k_2)^{-1} (c_{L0} - c_{D0}) [\exp(k_1 c_A t) - 1]\right].$$

Simple analysis of equations (4) reveals that if at the initial moment the system is racemic, i.e. $c_{L0} = c_{D0}$, then it will remain racemic all the time. If, however, the initial state of the system deviates from racemity (containing an excess of, say, L over D, $c_{L0} - c_{D0} > 0$), then the system will reach a monochiral terminal state (in which only the L form is present whereas the D species have completely vanished). Such a time-evolution of the Frank system will take place no matter how small the initial excess of one enantiomer is.

A systematic and non-stochastic (yet very small) deviation from racemity must occur in the case of all chiral molecules as a consequence of the action of weak nuclear forces (in particular, of weak neutral currents); details on this matter can be found elsewhere [2, 12, 16-21]. Model calculations show that for typical organic molecules, $(c_{L0} - c_{D0})/c_{L0} \sim 10^{-17}$.

The Frank Model with Variable c_A

In the case when c_A is not a constant, then the finding of explicit analytical expressions for the solution of the system (1) does not seem to be feasible. Nevertheless, it is possible to deduce the basic properties of this solution without knowing the actual forms of the functions $c_L(t)$ and $c_D(t)$.

First of all, instead of (2) we evidently have

$$c_L - c_D = (c_{L0} - c_{D0}) \exp\left[k_1 \int_0^t c_A dt\right] \quad (5)$$

Equation (5) enables us to draw conclusions about the behavior of the Frank model with variable c_A in the limit $t \rightarrow \infty$. Really, from the fact that for all values of t , the concentration $c_A(t)$ is necessarily non-negative, we see that for $t \rightarrow \infty$ the difference $c_L - c_D$ either (i) becomes unboundedly large or (ii) has a finite limit value. Evidently, case (ii) will occur only if the concentration of A vanishes for $t \rightarrow \infty$, or more precisely, if $\int_0^\infty c_A dt < \infty$.

In case (ii) $c_L - c_D \rightarrow \infty$ and consequently, $c_L \rightarrow \infty$. In view of the reaction (β) it must then be $c_D \rightarrow 0$. Thus we arrive at

the following result.

Theorem 1. If $c_A(t)$ is an arbitrary function such that $c_A(t) \geq 0$ for $t > 0$, and if the integral $I_\infty = \int_0^\infty c_A dt$ exists, then the Frank model has (i) a strictly monochiral terminal state, provided $I_\infty = \infty$, or (ii) a terminal state in which the two enantiomers are both present, but in unequal amounts, provided $I_\infty < \infty$. The excess of the dominant enantiomer exponentially increases with I_∞ .

On the Enantiomeric Excess in the Frank Model

The enantiomeric excess (*ee*), or chiral polarization of a system containing enantiomeric species L and D is defined as [4, 9, 14, 17]

$$ee = (c_L - c_D) / (c_L + c_D).$$

This dimensionless quantity has the convenient property that its value is zero for racemic, and unity for monochiral states of the system considered. In systems in which chemical reactions take place, *ee* is time-dependent. Its derivative with respect to *t* satisfies

$$d(ee)/dt = 2(c_D dc_L/dt - c_L dc_D/dt) / (c_L + c_D)^2 \quad (6)$$

Substituting equations (1) back into (6) we obtain

$$d(ee)/dt = 2 k_2 c_L c_D (c_L - c_D) / (c_L + c_D)^2$$

and by taking into account equation (5),

$$d(ee)/dt = 2 k_2 c_L c_D (c_{L0} - c_{D0}) \exp \left[k_1 \int_0^t c_A dt \right] / (c_L + c_D)^2$$

If $c_{L0} - c_{D0} > 0$, then the right-hand side of (7) is necessarily positive for all $t > 0$. We thus obtain the following result.

Theorem 2. If $c_A(t)$ is an arbitrary function, such that $c_A(t) \geq 0$ for $t > 0$, and if $c_{L0} - c_{D0} > 0$, then the enantiomeric excess monotonously increases for all $t > 0$.

Observe that the statement of Theorem 2 is independent of the actual numerical values of the parameters k_1 , k_2 , c_{L0} and c_{D0} . Also observe that Theorem 1 implies that for $t \rightarrow \infty$, the limit value of *ee* is either unity (if $I_\infty = \infty$), or less than unity (if $I_\infty < \infty$).

Conclusion

This paper shows that the basic properties of the Frank model are maintained for a very general class of functions

$c_A(t)$. Namely, a completely monochiral terminal state is achieved if and only if the integral $\int_0^t c_A dt$ unboundedly increases for $t \rightarrow \infty$. This, in particular, happens when c_A is a constant - as in the original formulation [11] of the Frank model. However, for the success of the Frank model the choice of constant c_A is not at all necessary. These features of the Frank model are established in this work for the first time.

In the case of time-dependent c_A , it is no more easy to find explicit expressions for the functions $c_L(t)$ and $c_D(t)$. The straightforward way to overcome this difficulty would be to find c_L , c_D and c_A by numerically solving the differential equations (1). For this, however, one would have to guess some concrete numerical values for the rate constants k_1 and k_2 as well as for the initial concentrations c_{L0} , c_{D0} and c_{A0} . Introduction of such ad hoc assumptions would certainly diminish the credibility of the Frank model, whose power lies in its simplicity and generality. This paper showed how this obstacle could be overcome by using mathematical analysis and how the basic properties of the Frank model could be established without knowing the actual analytical form of the solution of the respective system of differential equations and without referring to any particular numerical value of the rate constants and/or the initial concentrations.

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