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NUMERICAL STUDY OF NONLINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS BY ADOMIAN'S METHOD†

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Abstract

The main purpose of this paper is to consider Adomian's decomposition method in non-linear Volterra integro-differential equations. The advantages of this method, compared with the recent numerical techniques (in particular the implicitly linear collocation methods), and the convergence of Adomian's method applied to such nonlinear integro-differential equations are discussed. Finally, by using various examples, the accuracy of this method will be shown.

1. Introduction

Adomian's decomposition method [1-3] is a mathematical method which can be applied to the solution of linear or nonlinear differential equations, deterministic and stochastic operator equations, and many algebraic equations [2,11]. In this method, the solution is found as an infinite series which converge rapidly to accurate solution. (The most important work on convergence has been carried out by Y. Cherruault [4,5]). This method is well-studied for physical problems, since it makes unnecessary linearization, perturbation and other restricitve methods and assumptions which may change the problem, sometimes seriously. We know that the decomposition method can be considered as an extension to the successive approximation method, while being much more powerful [6]. Such a method is certainly efficient and easily computable.

Recently, Y. Cherruault and B. Some [5,7] applied the Adomian's method for Hammerstien nonlinear integral equations. In the present paper, we consider the Adomian's decomposition method for nonlinear Volterra integro-differential equations of the form

$$x'(t) = f(t, x(t)) + \int_0^t k(t, s)\, G(s, x(s))\, ds, \quad 0 \le t \le T \tag{1}$$

subject to the initial condition $x(0) = y$. In this equation, f and G are assumed smooth and known functions, and $f(t, v)$, $G(t, v)$ to be nonlinear in v. These equations have also been solved by different numerical techniques [12,15,16], which verify both the correctness of the solutions as well as the quickness of the Adomian's method.

We also study the convergence of Adomian's method applied to such nonlinear integro-differential equations. And finally, by using various examples, the accuracy of this method will be shown.

2. Preliminaries

Let E be a Banach space, and consider the general functional equaiton

$$y = Tx, \tag{2}$$

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where T is an operator from E into E, y is a given function in E, and we are looking for $x \in E$ satisfying (2). We assume that (2) has a unique solution for $x \in E$. Note that the Banach space E is not necessarily a finite-dimensional space, and it can be a functional space. Throughout this paper, we suppose E is a Banach space of L^2 functions.

We assume that in operator T, involving both the linear and nonlinear terms, the linear term is decomposed into $L+R$, where L is easily invertible and R is the remainder of the linear operator. L is taken as the highest order derivative avoiding the difficult integrations which result when complicated Green's functions are involved. The operator T is then decomposed: $T= L+R+N$, where N represents the nonlinear term.

Thus, Equation (2) is written:

$$y= Lx + Rx + Nx. \tag{3}$$

Then the solution x of (2) or (3) verifies

$$x= L^{-1}y - L^{-1}Rx - L^{-1}Nx, \tag{4}$$

where L^{-1} is the inverse of the linear operator L.

The Adomian's method [3,11] consists of representing x as a series,

$$x = \sum_{n=0}^{\infty} x_n .$$

The nonlinear operator is decomposed as

$$Nx = \sum_{n=0}^{\infty} A_n ,$$

where A_n represents the special polynomials of $x_0, x_1, \ldots, x_n$ defined by Adomian, that we obtain by writing

$$z = \sum_{i=0}^{\infty} \lambda^i x_i, \quad N\left(\sum_{i=0}^{\infty} \lambda^i x_i\right) = \sum_{i=1}^{\infty} \lambda^i A_i ,$$

with λ being a parameter introduced for convenience.

Definition. Adomian's Polynomials

Let N be an analytical function and Σx_n a convergent series in E. The Adomian polynomials are defined by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i x_i\right)\right]_{\lambda=0}, \quad n= 0,1,2,\ldots . \tag{5}$$

Generally, it is possible to obtain exactly A_n as a function of $x_0, x_1, \ldots, x_n$ from the nonlinearity N, see [6, pp. 47-48]. By these decompositions, formula (4) can be written

$$\sum_{n=0}^{\infty} x_n = L^{-1}y - L^{-1}R\left(\sum_{n=0}^{\infty} x_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{6}$$

Taking $x_0 = L^{-1}y$, we can identify the other terms of the series $\sum_{n=0}^{\infty} x_n$ by the following algorithm,

$$\begin{aligned} x_1 &= -L^{-1}Rx_0 - L^{-1}A_0 \\ x_2 &= -L^{-1}Rx_1 - L^{-1}A_1 \\ &\vdots \\ &\vdots \\ x_n &= -L^{-1}Rx_{n-1} - L^{-1}A_{n-1}. \end{aligned}$$

Thus, all components of x can be calculated once the A_n are given for $n= 0,1,2,\ldots$. Then we define n-term approximate to the solution, x by $\varphi_n(x)= \sum_{i=0}^{n-1} x_i$, with $\lim_{n\to\infty} \varphi_n(x) = x$. (For the basic concepts of the decomposition theory see [6]).

3. Adomian's Method Applied to Nonlinear Integro-Differential Equations

We consider a nonlinear Volterra integro-differential equation of the form

$$x'(t) = f(t, x(t)) + \int_0^t k(t, s)\, G(s, x(s))\, ds, \quad 0 \le t \le T \tag{7}$$

subject to the initial condition $x(0)= y$ in a Banach space of L^2 functions. In this equation, f is assumed smooth and k, G are L^2 integrable and known functions, and $f(t,v)$, $G(t,v)$ to be nonlinear in v.

This initial value problem may be written as

$$x(t) = y + \int_0^t f(s, x(s))\, ds + \int_0^t H(t, s)\, G(s, x(s))ds \tag{8}$$

where

$$H(t,s) = \int_s^t k(\tau, s)\, d\tau, \quad 0 \le s \le t \le T. \tag{9}$$

Equation (8) may be viewed as a special case of a generalization of the Volterra-Hammerstien integral equation.

If we assume that Equation (8) has the functional equation form, $Tx=y$, where

$$Tx(t) = x(t) - \int_0^t f(s, x(s))ds - \int_0^t H(t, s)G(s, x(s))ds, \tag{10}$$

then note that the operator T can be decomposed as: $T=L+N$, where $Lx=x$ is the linear term and $Nx = -\int_0^t f(s,x(s))ds - \int_0^t H(t, s)G(s, x(s))ds$ is the nonlinear term, with f and G nonlinear.

So (10) can be written as

$$Lx + Nx = y,$$

or

$$x + Nx = y. \tag{11}$$

According to Adomian's technique, the solution x of (11) is: $x = \sum_{n=0}^{\infty} x_n$, where the terms x_n are calculated by the following algorithm:

$$\begin{aligned} x_0 &= y \\ x_1 &= A_0 \\ x_2 &= A_1 \\ &\vdots \\ x_n &= A_{n-1}. \end{aligned}$$

For obtaining the Adomian's polynomials A_n, we have

$$N\left(\sum_{i=0}^{\infty} \lambda^i x_i\right) = -\int_0^t f\left(s, \sum_{i=0}^{\infty} \lambda^i x_i(s)\right) ds - \int_0^t H(t, s)\, G\left(s, \sum_{i=0}^{\infty} \lambda^i x_i(s)\right) ds = \sum_{n=0}^{\infty} \lambda^n A_n, \tag{12}$$

where λ is a parameter introduced for "convenience"

From (12) we obtain

$$n!\, A_n = \frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i x_i\right)\right]_{\lambda=0} = \frac{d^n}{d\lambda^n}\left[-\int_0^t f\left(s, \sum_{i=0}^{\infty} \lambda^i x_i(s)\right) ds - \int_0^t H(t, s)\, G\left(s, \sum_{i=0}^{\infty} \lambda^i x_i(s)\right) ds\right]_{\lambda=0}$$

and finally,

$$A_n = -\frac{1}{n!}\left[\int_0^t \frac{d^n}{d\lambda^n}[f(s, \sum_{i=0}^{\infty} \lambda^i x_i(s))\, ds\,]_{\lambda=0} + \int_0^t H(t,s) \frac{d^n}{d\lambda^n}[G(s, \sum_{i=0}^{\infty} \lambda^i x_i(s))ds]_{\lambda=0}\right]. \tag{13}$$

According to the above algorithm, we see that the solution x of Equation (8) can be determined by the calculation of $A_n s$, and we have

$$x = \sum_{n=0}^{\infty} x_n = y + \sum_{n=0}^{\infty} A_n.$$

Obviously, numercial computation of (13) is not expensive. (See section 5)

Remark. Application of projection methods (Galerkin and collocation methods) for (8) leads to a system of nonlinear equations. Usually, the numerical solution of these systems by iteration methods is complicated and expensive to implement, because the n definite integrals in nonlinear system need to be evaluated at each step of the iteration method. Kumar and Sloan [8] give a new collocation type method for numerical solution of Hammerstien integral equations. One of the advantages of this method, compared to the standard collocation method [16], is that the integrals which appear in the nonlinear system need to be calculated once only, and the result is a closed set of algebraic nonlinear equations for the n unknowns. H. Brunner [12] applied this method (which is referred to as the implicitly linear collocation method) to nonlinear Volterra integral equations. In section 5, by comparing the numerical solution of nonlinear integro-differential equations by Adomian's decomposition method and other numerical treatments cited above, we can see that Adomian's method avoids the cumbersome integrations of these methods.

4. Convergence of Adomian's Decomposition Method

In recent papers [4,5,6] Y. Cherruault, B. Some and L. Gabet gave theorems for convergence of the Adomian's decomposition method. Y. Cherruault [4] has given proof of the convergence of Adomian's method by using fixed point theorem. He introduced a new formulation of the method by setting $S_n = x_1+x_2+\ldots+x_n$ and proving that S_n is a solution of the fixed point equation

$$N(x_0+S)= S.$$

B. Some [7] gives a convergence theorem for application of Adomian's method to the Hammerstien integral equaiton, using the properties of the entire series substituted in another series. This theorem can be easily extended to nonlinear Volterra integral equations. If we do this substitution, and set $\lambda = 1$ because Nx can be developed in a Taylor series, we obtain an array or double series where i-row of this array converges to the A_i defined as in (13). (For details see [5]).

Now we consider the nonlinear Volterra integro-differential equation (7) and Adomian's polynomial (13), we also assume that f and G are analytical functions in $(0,T)$. Note that each A_n depends only on $x_0(t), x_1(t), \ldots, x_n(t)$. (See [5]). We prove that by hypothesis of [6], ΣA_n is a decomposition series which converges.

Let $x = \sum_{n=0}^{\infty} x_n$ and $z_x(\lambda) = \sum_{n=0}^{\infty} \lambda^n x_n$. ($\lambda$ being a real number). Obviously $z_x(\lambda)$, converges for $\lambda = 1$ and its sum is analytical over the open disc with center o, and radius $r(D(o,r))$, thus $z_x(\lambda)$ is analytical over $D(o,r)$. Since f and G are analytical, by using composition theorem we have $(f+G)\, o\, z_x(\lambda)$ is analytical over $D(o,r)$. Also note that $\lim_{\lambda\to 1} z_x(\lambda)=x$ and $\lim_{\lambda\to 1}(f+G) o\, z_x(\lambda)= f(s, x)+G(s,x)$. If the series $\sum x_n$ be convergent, then $\sum A_n$ converges and its sum is:

$$f\, o\, z_x(1) + G\, o\, z_x(1)= f(s, x) + G(s, x),$$

that is the decomposition series $\sum A_n$ weakly converges. Now suppose $\sum x_n$ and $\sum y_n$ are two series having the same sum x, and the Adomian's polynomials are respectively A_n^x, A_n^y, then we have

$$\sum_{n=0}^{\infty} A_n^x = f\, o\, z_x(1) + G\, o\, z_x(1)= f(s, x) + G(s, x)=$$

$$f\, o\, z_y(1) + G\, o\, z_y(1)= \sum_{n=0}^{\infty} A_n^y,$$

so the sum of the Adomian decomposition series depends only on the sum of the considered series, and the convergence is strong. Thus, we have the following theorem:

Theorem. If f and G are analytical functions of x in (0, T), then the Adomian's polynomial series ($\sum A_n$) in (13) for the nonlinear Volterra integro-differential Equation (7), define a decomposition series, that converges.

5. Numerical Results and Discussion

We consider four examples. The first one is linear integro-differential equation and the others are nonlinear. All computations were carried out on an IBM-PC using a program written in the symbolic language Mathematica, version 2.1 and long double precision. (Only Example 3 has been solved using single precision).

Example 1. Consider the linear integro-differential equation

$$x'(t)= 1 - 2t^3 e^{-t^2} + \int_0^t (2t - st^2)e^{-st} . x(s)\, ds, \quad 0 \le t \le 5$$

$$x(0)= 0$$

the exact solution of this problem is $x(t)= t$.

This initial value problem may be written as

$$x(t) = t - 1 + (1+t^2)e^{-t^2} + \int_0^t t^2 . e^{-ts} . x(s) ds, \quad 0 \le t \le 5$$

When applying Adomian's method, we have

$$x_0(t)= t - 1 + (1+t^2)e^{-t^2}$$

$$x_1(t) = A_0 = \int_0^t t^2 e^{-ts} . x_0(s)\, ds,$$

$$= [erf(\frac{3t}{2}) + erf(\frac{t}{2})] \frac{t^2(t^2+6)}{8} \sqrt{\pi} . e^{\frac{t^2}{4}}$$

$$+ (-t^2 + t - 1)e^{-t^2} - \frac{t^3 e^{-2t^2}}{4} - \frac{t^3 + 4t - 4}{4}$$

and so on .... ($erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-u^2} du$)

Table 1. Comparison between exact solution and approximate solution by two iterations of Adomian's method for Example 1

t	Exact solution	Adomian's method with two iterations
0.0	0.0	0.000000000
0.5	0.5	0.499434917
1.0	1.0	0.971420023
1.5	1.5	1.379273389
2.0	2.0	1.824789885
2.5	2.5	2.337918871
3.0	3.0	2.869041322
3.5	3.5	3.396446364
4.0	4.0	3.917792677
4.5	4.5	4.434179799
5.0	5.0	4.946823984

The approximate solution involving two terms is:

$$x(t)= x_0(t) + x_1(t)$$

Table 1 gives a comparison of our results and exact solution.

Example 2. From [16]

$$x'(t) = 1 + x(t) - te^{-t^2} - 2\int_0^t tse^{-x^2(s)}ds, \qquad 0 \le t \le 1$$

with exact solution $x(t)= t$.

We write this problem in the form

$$x(t) = t - \frac{1}{2} + \frac{1}{2}e^{-t^2} + \int_0^t x(s)\,ds -$$

$$2\int_0^t k(t,s)e^{-x^2(s)}\,ds, \qquad 0 \le t \le 1$$

where $k(t,s) = \frac{st^2}{2} - \frac{s^3}{2}$. After two iterations, Adomian's method gives the following results,

$$x_0(t) = t - \frac{1}{2} + \frac{1}{2}e^{-t^2}$$

$$x_1(t) = A_0 = \frac{\sqrt{\pi}}{4}erf(t) + \frac{t^9}{36} + \frac{t^8}{96} - \frac{2t^7}{35} + \frac{t^6}{12} + \frac{4t^5}{15} - \frac{t^4}{4} + \frac{t^2}{2} - \frac{t}{2}$$

and so on .... (Note that in this case and according to the method applied by Adomain, we approximate $e^{-x_0^2(s)} \simeq 1 - x_0^2(s)$. See [1, pp. 72]). Thus, the approximate solution by Adomian's method is:

$$x(t)= x_0(t) + x_1(t)$$

Table 2 gives a comparison between our results and those obtained by L.M. Delves and J.L. Mohamed [16].

Table 2. Comparison between the number of iterations of Adomian decomposition method and those obtained by L.M. Delves and J.L. Mohamed [16] for Example 2

t	Adomian's method with two iterations	Delves numerical results with h= 0.1	Number of iterations of Delves method
0.0	0.00000000,	0.00000000	0
0.1	0.09983641	0.10000029	2
0.2	0.19876723	0.20000293	2
0.3	0.29625674	0.30001155	3
0.4	0.39248725	0.40003417	3
0.5	0.48867642	0.50007946	3
0.6	0.58737415	0.60015711	3
0.7	0.69278871	0.70027967	4
0.8	0.81120202	0.80045807	4
0.9	0.95157721	0.90069319	5
1.0	1.12649013	1.00100103	7

Example 3.

$$x'(t) = \frac{2t^3}{3} - t^2 + 1 + \frac{t^3}{x(t)} - 2\int_0^t x^2(s)\,ds, \quad 0 \le t \le 1$$

with solution $x(t) = t$.

This problem may be written as,

$$x(t) = \frac{t^4}{6} - \frac{t^3}{3} + t + \int_0^t \frac{s^3}{x(s)}\,ds - 2\int_0^t k(t,s)x^2(s)\,ds, \; 0 \le t \le 1 \tag{14}$$

where $k(t, s) = t - s$.

Since the application of Galerkin and collocation methods for this problem can be quite expensive to implement [see section 3], we solve this problem by the implicitly linear collocation method. With notation of [12], we have

$$x(t) = \frac{t^4}{6} - \frac{t^3}{3} + t + \sum_{\mu=1}^{2}\int_0^t k_\mu(t, s)G_\mu(s, x(s))\,ds, \quad 0 \le t \le 1 \tag{15}$$

where

$$k_1(t, s) = s^3, \quad k_2(t, s) = -2(t - s)$$

$$G_1(t, x(t)) = \frac{1}{x(t)}, \quad G_2(t, x(t)) = x^2(t).$$

Let $z_r(t) = G_r(t, x(t))$ in (14), thus (15) has the implicitly linear form,

$$z_r(t) = G_r\left(t, \frac{t^4}{6} - \frac{t^3}{3} + t + \sum_{\mu=1}^{2}\int_0^t k_\mu(t, s)\, z_\mu(s)ds\right), r = 1,2 \tag{16}$$

i.e. we obtain a system of nonlinear integral equations. Suppose its solution $z(t) = (z_1(t), z_2(t))$, then the solution of (15) is given by

$$x(t) = \frac{t^4}{6} - \frac{t^3}{3} + t + \sum_{\mu=1}^{2}\int_0^t k_\mu(t, s)\, z_\mu(s)\,ds. \tag{17}$$

Each $z_r(t)$ ($r = 1,2$) is approximated by an element $\omega_r(t)$ in the spline space $S_1^{-1}(\Pi_N)$, such that,

$$\omega_r(t_n + \tau h_n) = \sum_{l=1}^{2} L_l(\tau)\, W_{n,l}^{(r)}, \qquad \tau \in [0,1], \; r = 1,2$$

where

$$L_l(\tau) = \prod_{k=1, k \ne l}^{2} \frac{(\tau - c_k)}{(c_l - c_k)}$$

denotes the *lth* Lagrange polynomial with respect to the collocation parameters $\{c_j\}$, and

$$\psi_n(t; \omega) = \frac{t^4}{6} - \frac{t^3}{3} + t + \sum_{\mu=1}^{2}\sum_{i=0}^{n-1} h_i \sum_{l=1}^{2} b_{i,l}^{(\mu)}(t)\, W_{i,l}^{(\mu)}, \tag{18}$$

$$W_{n,j}^{(r)} = G_r\left(t_{n,j}, \psi_n(t_{nj}; \omega) + h_n\sum_{\mu=1}^{2}\sum_{l=1}^{2} a_{n,l}^{(\mu)}(c_j)\, W_{n,l}^{(\mu)}\right), \quad r = 1,2. \tag{19}$$

The weights $\left\{a_{n,l}^{(\mu)}(\sigma)\right\}$ and $\left\{b_{i,l}^{(\mu)}(t)\right\}$ are defined as in (2.3d) and (2.3e) in [12], except that $k(t, s)$ is replaced by $k_\mu(t, s)$. ($\mu = 1,2$)

After computing the solution $\left\{W_{n,j}^{(r)}\right\}$ of the nonlinear system (19) using More and Consard's method [18], we have an approximation to x

$$V(t_n + \sigma h_n) = \psi_n(t_n + \sigma h_n; \omega) + h_n\sum_{\mu=1}^{2}\sum_{l=1}^{2} a_{n,l}^{(\mu)}(\sigma)\, \omega_{n,l}^{(\mu)} \qquad \sigma \in [0, 1]$$

that represents the desired approximation to the solution of (17). (For $N = 10$ we gave the numerical solution of (14) in Table 3).

On the other hand, if we apply Adomian's method with two iterations for (14), we have

$$\begin{aligned} x(t) = {} & 3.85123 \arctan(0.7699t - 1.28565) \\ & + 2.49907 \log(8.99996t^2 - 30.062152t + 40.29756) \\ & + 1.00185 \log(2.99999t + 4.02075) \\ & - 0.00061t^{10} + 0.003086t^9 - 0.00396t^8 - 0.0158730t^7 \\ & + 0.04444t^6 - 0.33333t^3 + t - 7.13016 \end{aligned}$$

Table 3 gives a comparison between Adomian's results and numerical solutions by the implicitly linear collocation method.

Example 4. From [10]

$$y'(t) = \int_0^t (t-s)^2 (t^2 - 2t(s+4) + s^2 + 8s + 12)e^{s-t}$$

Table 3. Comparison between approximate solutions by two iterations of Adomian method and the implicity linear collocation method for Example 3

t	Adomian's method with two iterations	Numerical results with N= 10
0.0	-0.002405	0.000181
0.1	0.097673	0.100342
0.2	0.197812	0.201544
0.3	0.298033	0.301611
0.4	0.398738	0.400798
0.5	0.500445	0.500094
0.6	0.604073	0.600192
0.7	0.710922	0.708522
0.8	0.822863	0.800267
0.9	0.942027	0.901924
1.0	1.071020	1.001887

$$\frac{y^4(s)}{1+2y^2(s)+2y^4(s)}\,ds, \qquad 0 \le t \le 10$$

$$y(0)=1$$

The theoretical solution is not known, but the exact solution at $x = 10$ is $y(10)= 1.25995582337$. This problem can be written as

$$y(t)=1+\int_0^t (t-s)^3(4-t+s)e^{s-t}\cdot\frac{y^4(s)}{1+2y^2(s)+2y^4(s)}\,ds, \qquad 0\le t\le 10$$

Adomian's method with four iterations gives:

$$y_0(t)=1$$

$$y_1(t)=A_0=\frac{t^4e^{-t}}{5}$$

$$y_2(t)=A_1=\frac{-4t^8(t-9)e^{-t}}{39375}$$

$$y_3(t)=A_2=\ldots$$

and so on .... Suppose $y(t)= y_0(t)+y_1(t)+y_2(t)+y_3(t)$, then we have

$$y(t)=1+\frac{t^4e^{-t}}{5}-\frac{4t^8(t-9)e^{-t}}{39375}+\ldots$$

and

$$y(10)\simeq 1.172477917$$

E. Hairer [10] has showed that this problem is equivalent to a 5-dimensional system of ordinary differential equations, and has solved this sytem. In Table 4, we compare our results with those of Hairer. (For simplicity, let $G(s)=\frac{y^4(s)}{1+2y^2(s)+2y^4(s)}$)

In all these cases (various types of nonlinearity), Adomian's technique gives very good results, and comparing to different numerical schemes, it is not expensive. Note that since a complicated term y in a nonlinear equation (8) can cause difficult integrations and proliferation of terms, we can expand y in a convenient series which is truncated, since in applied problems (physics or engineering problems, etc.), we only need accuracy to a certain number of decimal places. [cf. example 2]

6. Conclusion

In practice, we conclude that:

- The Adomian's method is a numerical elegant method that can solve various types of nonlinear integro-differential equations.
- This method avoids the cumbersome integrations of the recent numerical methods (Galerkin, collocation, implictly linear collocation, etc.) and in a few iterations gives very good results.
- Numerical computations of this method, compared to the numerical schemes are simple and inexpensive.
- The solution is given by a function, and not only at some grid points as in the projection methods.
- In comparison with the successvie approximation method, Adomian's decomposition method generally requires less computation.
- Convergence of the method is very fast.

Table 4. Absolute errors for our results and method of Hairer [10] for Example 4

	Absolute error
E. Hairer's results with 67 evaluations of $G(s)$	$7.76E-3$
Our results with 4 iterations	$8.74E-2$

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