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AXIAL FLOW IN A ROTATIONAL COAXIAL RHEOMETER SYSTEM 1. BINGHAM PLASTIC

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Abstract

A mathematical analysis has been carried out for the axial flow of a Bingham plastic fluid, in the Concentric Cylinder Viscometer which consists of a cylindrical sample holder (the cup) and a cylindrical spindle (the bob) coaxial with the cup. The fluid to be tested flows through the annular gap of the cup and the bob system, sheared by the rotation of the inner cylinder, while the outer cylinder is held stationary. This is a case of helical flow in an annular region. An attempt has been made to direct the analysis toward an examination of the relationship between moment (M) and angular velocity (Ω) at the inner cylinder. Since M and Ω are proportional to the shear stress and shear rate respectively, we may thus investigate the relationship between shear stress and shear rate and the dependence of this on axial flow rate. We have shown using numerical solutions that axial flow has no effect on shear rate. A definition for an approximate viscosity has been presented and a comparison has been made between predicted approximate and true viscosity.

Introduction

Rotational coaxial cup and bob rheometers are widely used in industry for the measurement of fluid viscosities. An on-line rotational viscometer (Figure 1) is sometimes favoured where the continuous measurement of viscosity of a process fluid is required. The flow pattern generated in an on-line rheometer could be of helical nature caused by the superposition of an axial flow on the rotational flow generated by the rotating component of the rheometer. The rotational motion in this rheometer may be considered as a simple shearing flow.

Theoretical analysis for helical flow was first considered by Rivlin [1]. Subsequently, the detailed formulation of the solution in the most general way was presented by several authors [2-5] considering unsteady state, laminar, tangential flow of an isothermal, incompressible viscous fluid in the annular space between two cylinders in which one or both might have been rotating. Fredrickson [6] suggested a meth(solve the problem of combined axial and tangential of Bingham plastic fluid in concentric cylinder an He also demonstrated how his equations coul generalised to give a solution to the problem pose Rivlin. Tanner [7] presented the theory of helical flo applied to a model due to Oldroyd [8] along with s experimental results. Specific helical flows have studied experimentally by some authors [9,10 number of other approaches to the solution of equation of motion for helical flow exists [11]. Rec Huilgol [12] solved the helical flow for general flui terms of four parameters including one related to pres drop along the axial flow. He proposed a trial and method to solve his equations. No theoretical result his analysis is available at this stage. Bhattacharya [13] solved the problem of axial flow in a rotati rheometer for power law fluids. Their solution obtained in terms of three parameters, i.e. torque per

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length, axial flow and angular velocity and the two parameters for the fluid model. They compared experimental results with the predicted values from their theoretical analysis and observed good agreement.

The aim of this paper is to extend the analysis for helical flow of a Bingham plastic fluid in an annular space.

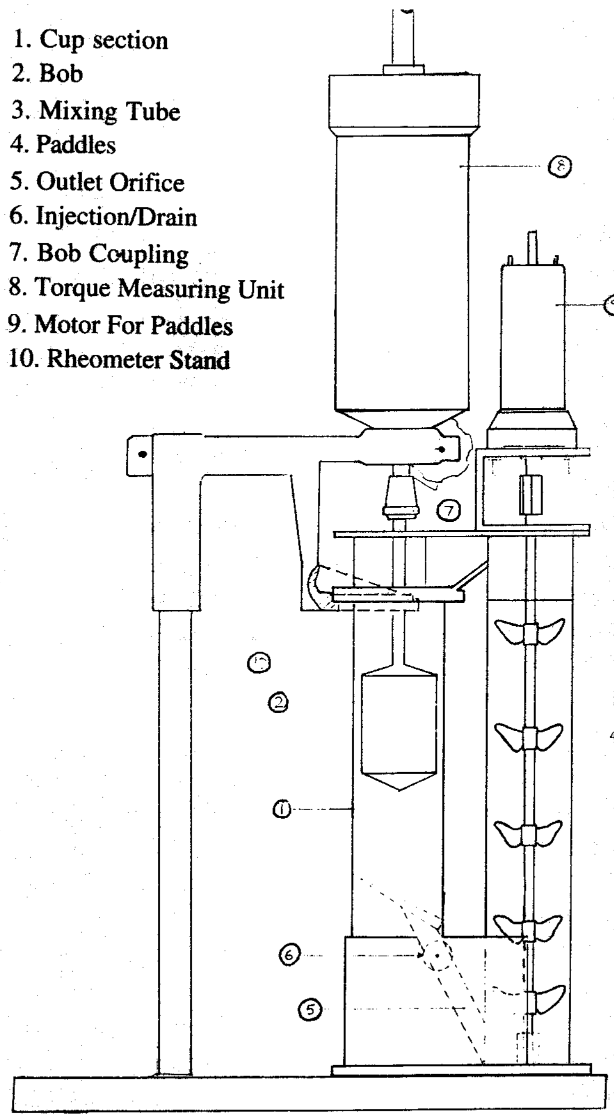

Figure. 1. Axial Flow Rheometer

Mathematical Model

We consider the steady helical flow of a simple fluid in the annular space between two coaxial circular cylinders. We use cylindrical coordinates

$$\chi^1 = z, \chi^2 = r, \chi^3 = \theta \quad (1)$$

We envision such a flow to be generated by imposition of an axial flow, characterized by Q, and by steady rotation of the inner cylinder (or bob of radius R_1) of finite length rotated with angular velocity Ω. The shear stress induced by this rotation gives rise to a moment M, experienced at the inner cylinder, and measurable there. Fundamental to such a device is a relationship between M, Ω and the physical parameters of the fluid; it is not clear how the M versus Ω relationship is likely to be affected once an axial flow, Q is introduced. We will attempt to clarify this matter.

This important class of flow which is admitted by virtue of our broadened classification is that which Coleman and others have called "Curvilineal flow". These are detined by the criterion that contravariant components of the velocity field have the form

$$\underline{V} = (v(r), O, r\omega(r)) \quad (2)$$

which automatically satisfies the equation of continuity.

We express the gravitational force in terms of a scalar potential ψ by

$$g = -\nabla\psi \quad (3)$$

and define

$$\Phi = P + \rho\psi \quad (4)$$

where ρ is the fluid density (constant), P is the fluid pressure. Thus the components of the equation of motion, written in terms of physical components of the stress tensor are

$$\frac{1}{r}\frac{d}{dr}(rt_{(rr)}) - \frac{1}{r}t_{(\theta\theta)} - \frac{\delta\Phi}{\delta r} = -\rho r\omega^2 \quad (5)$$

$$\frac{1}{r}\frac{d}{dr}(rt_{(rz)}) - \frac{\delta\Phi}{\delta z} = 0 \quad (6)$$

$$\frac{1}{r}\frac{d}{dr}(r^2 t_{(r\epsilon)}) - \frac{\delta\Phi}{\delta\theta} = 0 \quad (7)$$

where $t_{(rr)}, t_{(r\theta)}, t_{(rz)}, t_{(\theta\theta)}$, are the usual components of the stress tensor.

In the present example, we obtain

$$t_{(rz)} = \frac{v'}{\xi}\tau(\xi) \quad ; \quad t_{(r\theta)} = \frac{r\omega'}{\xi}\tau(\xi) \quad (8)$$

where τ the shear stress, will depend on the local rate of shearing, ξ, given by

$$\xi = \sqrt{(v')^2 + (r\omega')^2} \quad (9)$$

We know from (2) and our postulate of a simple fluid that the stress can only be a function of r. This leads to the conclusion that Φ can at most be of the form

$$\Phi = g(r) + 2\alpha_1 z + c_1\theta \quad (10)$$

where α and c_1 are constants. Then integration of (6) and (7) results in

$$t_{(rz)} = \alpha r + \frac{1}{r}\beta \quad (11)$$

$$t_{(r\theta)} = \frac{1}{2} c_1 + \frac{1}{r^2} c_2 \qquad (12)$$

where the constants are determined by application of suitable boundary conditions.

From a combination of (8) with (11) and (12) there results a pair of differential equations for the velocity components

$$v'(r) = \left(\alpha r + \frac{\beta}{r}\right) \frac{\xi}{\tau(\xi)} \qquad (13)$$

$$\omega'(r) = \left(\frac{1}{2r} c_1 + \frac{c_2}{r^3}\right) \frac{\xi}{\tau(\xi)} \qquad (14)$$

where from (9) one may write

$$\tau(\xi) = \left\{\left(\alpha r + \frac{\beta}{r}\right)^2 + \left(\frac{c_1}{2} + \frac{c_2}{r^2}\right)^2\right\}^{\frac{1}{2}} = f(r) \qquad (15)$$

$$\xi = \tau^{-1}(f(r)) \qquad (16)$$

These relationships are well-documented in the literature, for example see [14].

Thus in principle we can solve for the velocity profile from (13)-(14) once the boundary conditions, and hence the values of c_1, c_2, α, and β are known. We see that the velocity field is completely determined, for a given set of boundary conditions, by the single material function $\tau(\xi)$.

The non-slip condition applied at the inner and outer cylinders yield the boundary conditions

$$v(R_1) = v(R_2) = 0, \qquad (17)$$

$$\omega(R_1) = \Omega, \quad \omega(R_2) = 0 \qquad (18)$$

where Ω is the angular velocity of rotation of the inner cylinder.

Analysis for a Bingham Plastic Fluid

We now consider the situation where the relationship between τ and ξ is as follows:

$$\tau(\xi) = \delta_y + \mu_p \xi \text{ and } \eta(\xi) = \mu_p + \delta_y \xi^{-1} \qquad (19)$$

where $\eta(\xi)$ is known as the shear-dependent viscosity or "apparent viscosity", δ_y is the yield stress and μ_p is the plastic viscosity.

Equations (13) and (14) then become

$$\left[\mu_p + \delta_y (r^2 \omega'^2 + v'^2)^{-\frac{1}{2}}\right] v'(r) = \alpha r + \frac{\beta}{r} \qquad (20)$$

$$\left[\mu_p + \delta_y (r^2 \omega'^2 + v'^2)^{-\frac{1}{2}}\right] r\,\omega'(r) = -\frac{M}{2\pi r^2} \qquad (21)$$

Equations (20), (21) together imply

$$v'(r) = -\frac{2\pi}{M} (\alpha r^2 + \beta) r^2 \omega' \qquad (22)$$

Substituting equation (22) into (21) and noting that $\omega'<0$ everywhere, since the inner cylinder rotates and the outer one is stationary

$$\omega'(r) = \frac{M}{2\pi\mu_p} r^{-3} - \psi(r, \alpha, \beta) \qquad (23)$$

where

$$\psi(r, \alpha, \beta) = +\frac{\delta_y}{\mu_p} r^{-1} \left\{1 + \frac{4\pi^2 r^2}{M^2} (\alpha r^2 + \beta)^2\right\}^{-\frac{1}{2}} \qquad (24)$$

and integration of (23) subject to the boundary conditions (18) gives

$$\Omega = \frac{M}{4\pi\mu_p} (R_1^{-2} - R_2^{-2}) + \int_{R_1}^{R_2} \psi(r, \alpha, \beta)\, dr \qquad (25)$$

an equation linking Ω and M through α and β.

Applying equation (23) to (22) and integrating subject to the condition (17) yields

$$G_1(\alpha, \beta) = \frac{1}{\mu_p} \int_{R_1}^{R_2} (\alpha r^2 + \beta) r^{-1}\, dr + \frac{2\pi}{M} \int_{R_1}^{R_2} (\alpha r^2 + \beta) r^2 \psi(r,\alpha,\beta)\, dr = 0 \qquad (26)$$

However, there is a further constraint on the axial velocity v (r) that arises from the assumption of incompressible flow, i.e the axial flow rate at any cross-section of the annulus is constant.

If Q is this constant rate, this implies the extra condition

$$Q = 2\pi \int_{R_1}^{R_2} r v(r) dr \qquad (27)$$

Integrating by parts using (17) we obtain

$$\pi \int_{R_1}^{R_2} r^2 v'(r) dr + Q = 0 \qquad (28)$$

Equations (28), (22) and (23) together give

$$G_2(\alpha, \beta) = \frac{M}{2\pi\mu_p} \int_{R_1}^{R_2} (\alpha r^2 + \beta) r\, dr + \int_{R_1}^{R_2} (\alpha r^2 + \beta) r^4 \psi(r, \alpha, \beta) dr + \frac{MQ}{2\pi^2} = 0 \qquad (29)$$

We may now regard the problem as one of finding α and β from the simultaneous nonlinear equations

$$G_1(\alpha, \beta) = G_2(\alpha, \beta) = 0 \qquad (30)$$

This will allow us to obtain the function $v(r)$, $\omega(r)$ and hence the complete velocity field. More significantly for the present discussion α and β may be substituted into (25) to establish a relationship linking Ω and M with Q appearing as a parameter, via equation (29).

There are two special cases of the above analysis that deserve particular comment.

(i) For an incompressible Newtonian fluid $\delta_y = 0$ and $\mu_p = \mu_o$, the constant fluid viscosity, Then equations (26) and (29) become

$$\frac{1}{2}(R_2^2 - R_1^2)\,\alpha + \beta \ln\left(\frac{R_2}{R_1}\right) = 0 \qquad (31)$$

and

$$\frac{1}{4}(R_2^4 - R_1^4)\,\alpha + \frac{1}{2}\beta(R_2^2 - R_1^2) + \frac{\mu_o Q}{\pi} = 0 \qquad (32)$$

respectively, providing a relationship of α and β in terms of Q. However, since $\psi(r, \alpha, \beta) \equiv 0$, the relationship (25) is then independent of α and β, and hence Q as follows

$$\Omega = \frac{M}{4\pi\mu_o}(R_1^{-2} - R_2^{-2}) \qquad (33)$$

which is our basic relationship linking Ω and M.

(ii) With no axial flow $v \equiv 0$, and $Q = 0$, Equation (21) reduces to

$$r\omega' = -\frac{1}{\mu_p}\left|\delta_y + \frac{M}{2\pi r^2}\right| \qquad (34)$$

This equation holds in the region $R_y < r < R_2$ where R_y is given by

$$\delta_y = \frac{M}{2\pi R_y^2} \qquad (35)$$

If $R_y \le R_1$, particles in the fluids have the same angular velocity as they have at radius R_y and the material rotates as a rigid body with the inner cylinder. The relationship between Ω and M depends on whether R_1 is greater or less than R_y.

If $R_1 < R_y \le R_2$, some material near the outer cylinder will not be sheared, therefore

$$\Omega = \frac{M}{4\pi\mu_p}(R_y^{-2} - R_2^{-2}) + \frac{\delta_y}{\mu_p}\ln\frac{R_2}{R_y} \qquad (36)$$

we have plastic flow for $R_1 < r < R_y$ and solid like behaviour for $R_y \le r \le R_2$

Substituting for R_y from (35) we get

$$\Omega = \frac{\delta y}{2\mu_p}\left[1 - \ln\frac{M}{2\pi R_2^2 \delta y}\right] - \frac{M}{4\pi\mu_p R_2^2} \qquad (37)$$

If $R_y \ge R_2$ so that all the material is sheared

$$\Omega\ \frac{M}{4\pi\mu_p}(R_1^{-2} - R_2^{-2}) + \frac{\delta_y}{\mu_p}\ln\frac{R_2}{R_1} \qquad (38)$$

which reduces to (33) when $\delta_y = 0$ and $\mu_p = \mu_o$ as we would expect.

Estimation of Shear Stress and Viscosity

In the present problem non-zero components of the stress tensor are shown in equation (8), where $\tau_{(rz)}$ is the shear stress in the z direction and $\tau_{(r\theta)}$ is the shear stress in the θ direction. The total shear stress in the z - θ plane may be defined as

$$\tau = \sqrt{\tau_{(r\theta)}^2 + \tau_{(rz)}^2} \qquad (39)$$

Based on the local rate of shearing ξ given by (9), viscosity may be obtained as shear stress divided by the shear rate, i.e.

$$\eta = \frac{\sqrt{\tau_{(r\theta)}^2 + \tau_{(rz)}^2}}{\sqrt{v'^2 + (r\omega')^2}} \qquad (40)$$

which is often called the effective viscosity.

Viscosity can be calculated from the relations

$$\tau_{(rz)} = \eta\,(\tau \text{ or } \xi)\,v',\quad \tau_{(r\theta)} = \eta\,(\tau \text{ or } \xi)\,r\omega' \qquad (41)$$

Fluidity Function

The total shear stress $\tau\ (\xi)$ may be defined as

$$\tau(\xi) = \eta(\xi)\,\xi \qquad (42)$$

Assuming that τ is a single valued function of ξ, we can define the inverse function of viscosity as fluidity function $\phi\ (\tau)$, [15], so that

$$\xi = \tau\,\phi(\tau) \qquad (43)$$

After using equation (19) and (39) we obtain

$$\eta = \mu_p\left[1 + \frac{\delta_y}{\sqrt{\tau_{(r\theta)}^2 + \tau_{(rz)}^2} - \delta_y}\right] \qquad (44)$$

We can also find viscosity by substituting equation (9) into (46) as

$$\eta = \mu_p + \frac{\delta_y}{\sqrt{v'^2 + (r\omega')^2}} \qquad (45)$$

Definition of Approximate Rate of Shear and Viscosity

As a special case of the helical flow, we may obtain the Poiseuille flow and torsional flow, if we set $\omega = 0$ in the first case or $v = 0$ in the second.

In the narrow-gap Couette rheometer, the shear rate ξ_1 can be effectively taken as constant throughout the liquid, given by

$$\xi_1 = \frac{\Omega R_1}{R_2 - R_1} \quad (46)$$

where Ω is the rotational speed and R_1, R_2 are the inner and the outer cylinder radii respectively.

In the case of tube flow (Poiseuille flow), the shear rate ξ_2 can be taken as

$$\xi_2 = \frac{4v}{R_2 - R_1} \quad (47)$$

where

$$v = \frac{Q}{\pi (R_2^2 - R_1^2)} \quad (48)$$

We define the approximate rate of shear as

$$\xi_{app.} = \sqrt{\xi_1^2 + \xi_2^2} \quad (49)$$

Therefore the approximate viscosity is

$$\eta_{app.} = \mu_p + \delta_y \xi_{app.}^{-1} \quad (50)$$

Numerical Solution

The values of α and β of equation (30) are found by a simple numerical procedure which uses the secant method to solve

$$G_1 (\alpha(\beta), \beta) = 0 \quad (51)$$

where for each value of β, the value of $\alpha(\beta)$ is found by solving the other non-linear equation

$$G_2 (\alpha, \beta) = 0 \quad (52)$$

by further use of the secant method.

Specifically, a sequence of approximation β_2, β_3, ... is generated from initial estimates β_0 and β_1 using

$$\beta_{i+2} = \beta_{i+1} - \frac{G_1(\tilde{\alpha}(\beta_{i+1}), \beta_{i+1})(\beta_{i+1} - \beta_i)}{G_1(\tilde{\alpha}(\beta_{i+1}), \beta_{i+1}) - G_1(\tilde{\alpha}(\beta_i), \beta_i)} \quad (53)$$

for i = 1, 2, ... where $\tilde{\alpha}(\beta_i)$ is an estimate of $\alpha(\beta_i)$, the limit of the sequence $\alpha_{i,2}$, $\alpha_{i,3}$,... generated by

$$\alpha_{i,j+2} = \alpha_{i,j+1} - \frac{G_2(\alpha_{i,j+1}, \beta_i)(\alpha_{i,j+1} - \alpha_{i,j})}{G_2(\alpha_{i,j+1}, \beta_i) - G_2(\alpha_{i,j}, \beta_i)} \quad (54)$$

for j = 0,1, 2, ...

This procedure proved to be robust to the choice of initial estimates of α and β and was found to be rapidly convergent. Consequently, a common initial value may be used for $\alpha_{i,1}$, i = 1, 2,... and $\alpha_{i,2}$, i = 1, 2, ... without significant increase in the computational effort needed to find high accuracy estimates of α and β.

The integrations needed to evaluate G_1 and G_2 were carried out using the IMSL Version 10 adaptive integration routine QDAGS. Since the integrands have no exceptional features, little difficulty was experienced in achieving high accuracy with this routine. These numerical integrations were not a significant source of error in the estimation of α and β.

We calculated the values of α and β by substituting these values into equation (11). We find that there is a point r =R such that $R_1 < R < R_2$ where $\tau_{(rz)} = 0$ which means that [12]

$$R_1^2 < -\frac{\beta}{\alpha} < R_2^2$$

Clearly this provides a simple check on our calculation. Also the values of α and β obtained in such computation could now be applied to the equations (40), (41), (44), and (45) for which the values for viscosity can be estimated in various ways. We found the same value for viscosity from different relationships.

Alternatively, and more significantly for the present discussion, the equation may be applied to examine the relationship between shear-rate and shear stress at the inner cylinder $r = R_1$ and also at the different radial points in the annular region.

Finally, the values of α and β obtained from computations could be applied directly to the developed mathematical models to obtain the function v and ω via equations (22) and (23), and hence the velocity profile (2) by further computation.

Results and Discussion

Equation (30) has been solved to provide a relationship between the parameters M and Ω. M can be related to stress, τ by the following relation

$$\tau = \frac{M}{2\pi R_1^2} \quad (55)$$

where τ is the shear stress at the inner cylinder.

Theoretical results of shear stress versus shear rate have been obtained for different axial velocities. In Figure 2 these theoretical predictions for the axial flow rates of 1×10^{-6}, 10×10^{-6}, and 100×10^{-6} m^3/s are given, assuming the Bingham plastic parameters as $\mu_p = 2$ Pas and $\delta_y = 5$ Pa. It is interesting to note that the axial velocity does not have any effect on the shear

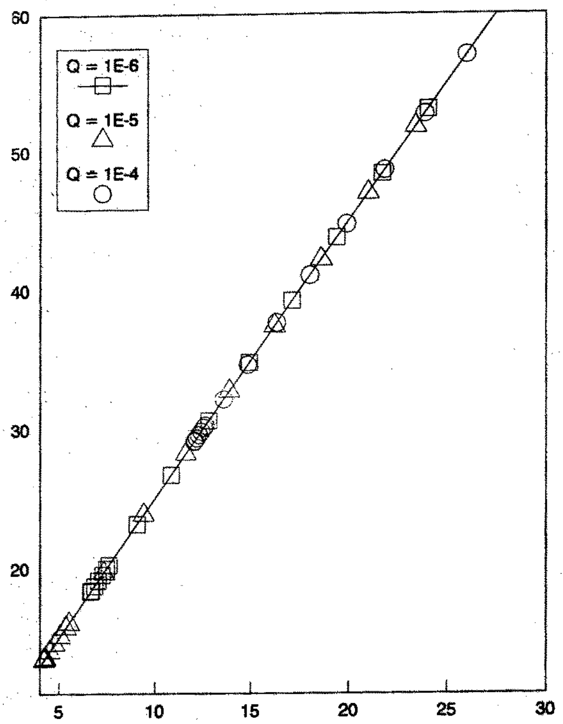

Figure.2. Effect of axial flow on shear stress.
$\mu_p = 2.0$ Pas, $\delta_y = 5$ Pa
Shear stress (Pa) Shear rates (s-1)

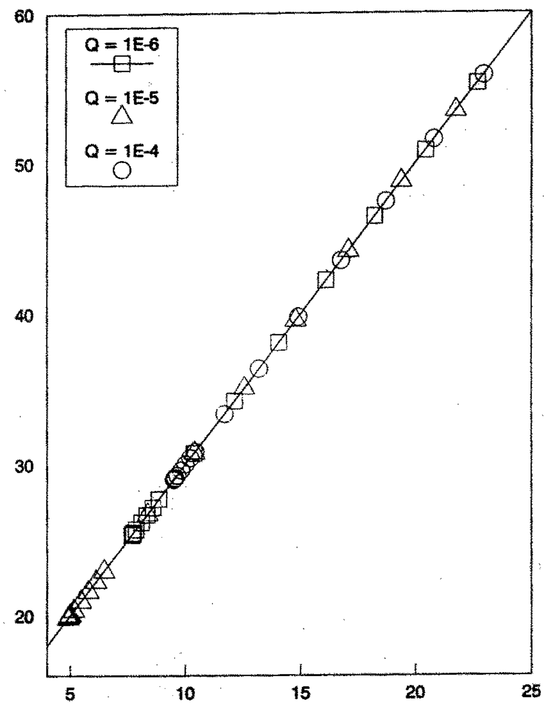

Figure.3. Effect of axial flow on shear stress.
$\mu_p = 2.0$ Pas, $\delta_y = 10$ Pa
Shear stress (Pa) Shear rates (s-1)

stress versus shear rate relation. Similar behaviour was observed as noted in Figure 3 when the yield stress(δ_y) was increased to 10 Pa.

It is recognised that in a Couette flow the shear stress varies with the radial position within the annular gap. Variation of viscosity and shear stress with radial position is given respectively in Figures 4 and 5. The point of minimum shear stress and maximum viscosity appears to be closer to the inner cylinder than the outer cylinder. A significant variation of the magnitude of these variables with radial position is also noted. It is obvious that the viscosity measurement should be consistently carried out at a particular radial position, which in this case is the surface of the inner cylinder. The estimated viscosity at the surface of the inner cylinder for $Q = 1 \times 10^{-6}\ m^3/s$ is presented in Figure 6. As expected the viscosity data is identical to that estimated by substituting $\mu_p = 2$ Pas and $\delta_y = 5$ Pa in equation (19).

An approximate value of the viscosity has been calculated using equation (50). This prediction is simple and may be used to compare with the rigourous prediction presented in Figures 2 to 6. A comparison between these approximate values and the rigourous or 'true' values is presented in Figure 7 and 8. The difference between the true and the approximate values is reduced when shear rate increases. Moreover this difference is larger at the inner surface than at the outer surface. However, the trend of the result indicates that at high shear rate the difference between true and approximate viscosity approaches zero.

In conclusion, this analysis shows that the axial flow rate does not have any effect on the viscosity measurement in a rotational coaxial rheometer.

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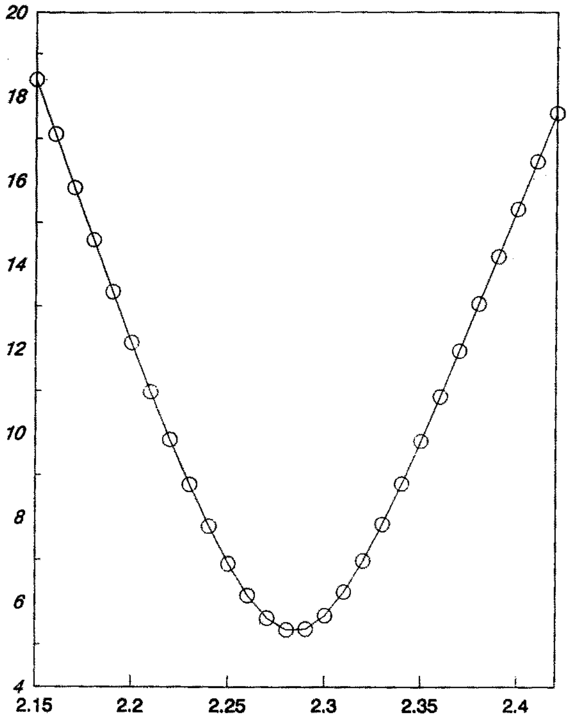

ure. 4. Variation of shear stress in the radial direction of the annular gap

R_1 = 0.0215, R_2= 0.0242, μ_p = 2.0 Pas, δ_y = 5.0 Pa and Q = 10^{-6} m/s

r stress (Pa) Radial position, m (Times 1E-2)

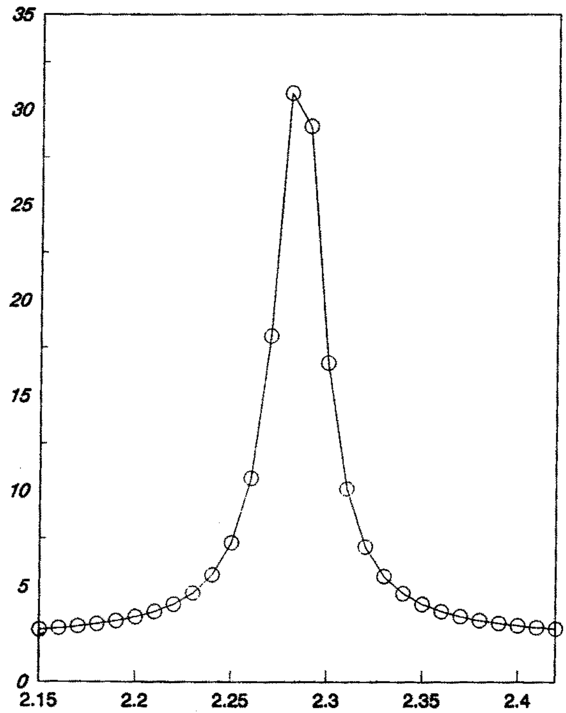

Figure. 5. Variation of viscosity in the radial direction of the annular gap

R_1 = 0.0215, R_2= 0.0242, μ_p = 2.0 Pas, δ_y = 5.0 Pa and Q = 10^{-6} m/s

Viscosity, (Pas) Radial position, m (Times 1E-2)

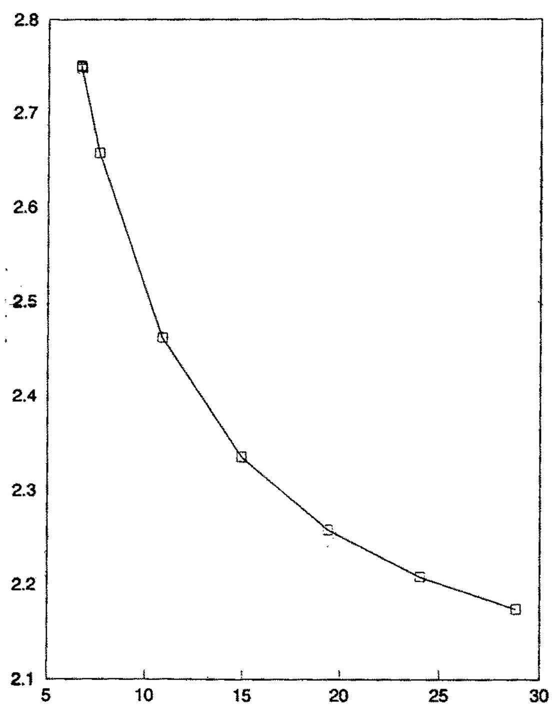

Figure.6. Estimated viscosity at the bob surface.

μ_p = 2.0 Pas, δ_y = 5 Pa and Q = 10^{-6} m^3/s

Viscosity (Pas) Shear rate, (s-1)

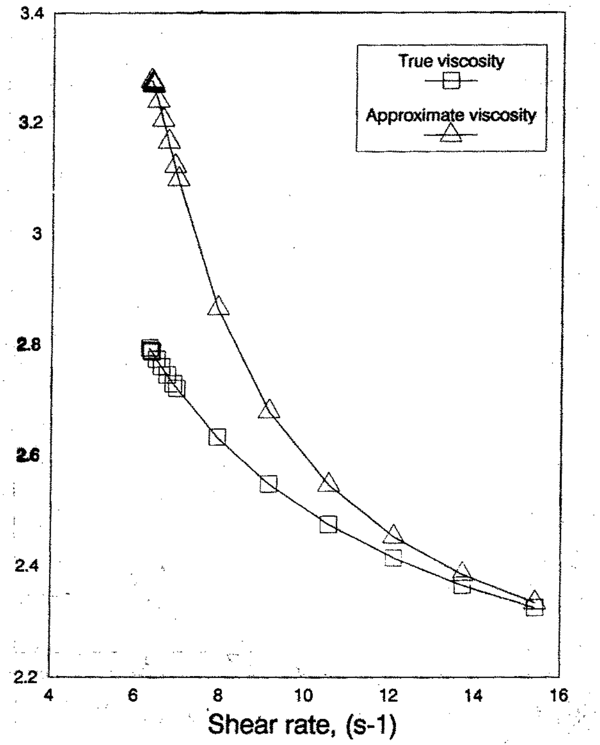

Figure 7. Comparison between true and approx. viscosity at $R_2 = 0.0242$ m

$\mu_p = 2.0$ Pas, $\delta_y = 5$ Pa and $Q = 10^{-6}\ m^3/s$

True viscosity

Approximate viscosity

Viscosity (Pas) Shear rate, (s-1)

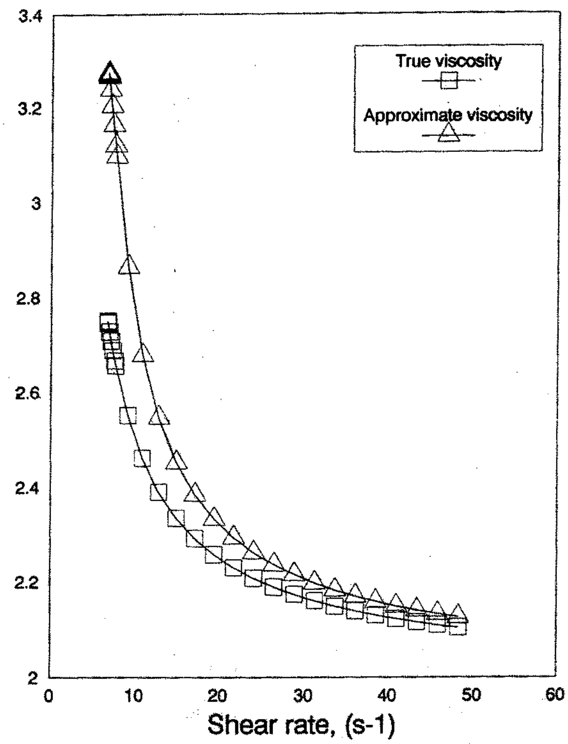

Figure 8. Comparison between true and approx. viscosity at $R_1 = 0.0215$ m

$\mu_p = 2.0$ Pas, $\delta_y = 5$ Pa and $Q = 10^{-6}\ m^3/s$

True viscosity

Approximate viscosity

Viscosity (Pas) Shear rate, (s-1)

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