

THE EFFECT OF COSMIONS ON THE STABILITY OF MAIN SEQUENCE STELLAR CORES

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Abstract

We have studied the effect of hypothetical Cosmions on the core stability of main sequence stars (of populations I and II). Cosmions, with a mass of 4-10 GeV/c² and a scattering cross section with nucleons of approximately 10⁻³⁶ cm² could prevail in transporting heat in the stellar cores. Raby [17] showed the existence of a local thermal instability caused by the presence of Cosmions in the solar core. Here we have used more accurate analytic relations and have considered all of the main sequence stars which have captured an efficient number of these hypothetical particles from their birth up to the present time. We have found a wide range of probable instabilities in the stellar cores for various values of Knudsen number, cosmion mass and cosmion cross section.

Introduction

The Cosmion, a special kind of Weakly Interacting Massive Particle (WIMP), has, on one hand, been proposed as a candidate for dark matter and, on the other hand, as a solution to the solar neutrino problem [2,8,9,10,14,21][†].

These particles, assumed to constitute the galactic dark halo, are captured by the sun and other stars,

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including main sequence stars. With a relative number density of about 10⁻¹¹ in the sun, they can transfer a

large amount of heat and decrease the central temperature to the extent that the computed flux of generated solar neutrinos agrees with the observed flux [4,16].

Cosmions, moreover, may have other effects on the structure and evolution of stars, including the suppression of stellar core convection and, consequently, through depriving the nuclear burning region from fresh fuel, bring about the premature death of a large number of stars, particularly low-mass main sequence stars [5].

Cosmions could also cause a local thermal instability in the solar core as studied by Raby [17]. They may also help to solve the age problem [20].

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[†] The authors are aware of some research findings according to which the differences between the real Sun and the standard model Sun are small. For example, recent work by Bahcall, Basu and Kumar on helioseismology shows that "the sound speeds of the real and the standard model Suns differ by less than 0.3% for regions of radial width ~0.1 R_☉ in the solar core" (see [2]). But it must be said that 1- the solar neutrino problem is not yet solved; 2- the galactic halo contains a lot of dark matter for which the Weakly Interacting Massive Particles [WIMPs] are good candidates; and 3- it is possible to consider a scenario in which a special kind of the WIMPs, called Cosmion, may solve the solar neutrino problem. We hope the difference between the real Sun and the Sun according to scenarios like this would be less than difference between the real Sun and the standard model Sun.

Many aspects of cosmions have been investigated thoroughly, including their mass, cross section, density and velocity in the galactic disc and halo, number density in the stars, distribution [4,12,13,16], their candidates [14,18], their evaporation [11] and their pair annihilation [4] and so on. We study the effect of cosmions on the core stability of main sequence stars of population I in the mass range $0.24M_{\odot} < M_* < 20M_{\odot}$ and of population II in the mass range $0.24M_{\odot} < M_* < 1.0M_{\odot}$. In this investigation we use linear instability analysis offered by Baker [3] and used by Clayton [6] and Raby [17].

In the next section, we present a brief discussion of the structure of main sequence stars. This includes convenient expressions for the temperature, density, pressure, entropy, energy generation rate, opacity, main sequence lifespan, chemical composition and mean molecular weight of stellar cores.

The scope of the research is discussed in the following section. The limits of this scope are set by temperature and pressure of the stars, on the one hand, and the number of captured cosmions on the other hand.

Next, the equations of stellar structure are perturbed due to the presence and effect of cosmions. A third order algebraic equation is derived, the sign of its real solution and the real part of its complex solution indicating the stability or instability of the stellar core.

Then we solve the equation for the stars with both negligible radiation pressure and electron degeneracy pressure.

Finally, we present the numerical results as well as a summary of the instability modes in the final section.

The Structure of Main Sequence Stars

The basic equations of stellar structure are as follows [6]:

$$\frac{dr}{dM} = \frac{1}{4\pi r^2 \rho(P,T)} \quad (1)$$

$$\frac{dP}{dM} = -\frac{GM}{4\pi r^4} - \frac{1}{4\pi r^2} \frac{dr^2}{dt^2} \quad (2)$$

$$\frac{dL}{dM} = \epsilon - T \frac{dS}{dt} \quad (3)$$

$$\frac{dT}{dM} = -\frac{3\kappa}{4acT^3} \frac{L}{16\pi^2 r^4} \quad (4)$$

in which the usual physical quantities and parameters have been used. Main sequence stars burn hydrogen. The chemical composition of POP I and POP II stars are typically:

$$\text{POP I: } X=0.70, Y=0.28, Z=0.02$$

$$\text{POP II: } X=0.76, Y=0.24, Z=0.002$$

The mass fraction of C, N and O in both cases is approximately $Z_{ONC} = \frac{Z}{4}$. In the following section we consider convenient expressions for temperature, density, pressure, entropy, energy generation rate, opacity, main sequence lifespan of stars, chemical composition and mean molecular weight of main sequence stellar cores.

Temperature and Density

For central temperature and density, in the absence of cosmions, we use the relations given by Bouquet and Salati [4]:

$$T_c = 1.5 \times 10^7 \left(\frac{M}{M_{\odot}}\right)^{0.4} \text{ } ^{\circ}\text{K} \quad (5)$$

$$\rho_c = 150 \left(\frac{M}{M_{\odot}}\right)^{-0.8} \text{ gr.cm}^{-3} \quad (6)$$

The presence of cosmions causes the temperature of the stellar core to decrease. In contrast, the central density of the stars does not change appreciably. In order to calculate the central temperature in the presence of cosmions, we use the polytropic expansion of the temperature [7]:

$$T(r) \approx T_c \left[1 - \frac{1}{6} \left(\frac{\xi_1 r}{R}\right)^2 + \frac{1}{40} \left(\frac{\xi_1 r}{R}\right)^4 - \dots \right] \quad (7)$$

where $\xi_1=6.8985$ is the first zero of Emden variable in the standard $n=3$ polytropic model and R is the stellar radius. Since the cosmionic heat transfer brings about an isothermal core and the temperature-radius curve will be almost flat through the region where cosmions effectively transport heat, the temperature of the stellar core can be approximately taken as the unperturbed temperature at r_w (the cosmion scale height) given by:

$$r_w = \left(\frac{9}{4\pi} \frac{kT_w}{G\rho_c m_w} \right)^{\frac{1}{2}} \quad (8)$$

We neglect the fourth and higher order terms in Equation (7). Furthermore, we use the following relation between the radius and the mass of a star [4]:

$$\frac{R}{R_{\odot}} = \left(\frac{M}{M_{\odot}} \right)^{0.8} \quad (9)$$

Now from Equations (5-9) we get:

$$T_w(0) = 15 \times 10^6 \left(\frac{M}{M_{\odot}}\right)^{0.4} \left[1 + 0.144 \left(\frac{M}{M_{\odot}}\right)^{-0.4} \frac{m_p}{m_w} \right]^{-1} \text{ } ^{\circ}\text{K} \quad (10)$$

where m_p is the proton mass.

Pressure

In general, there are three kinds of pressure in stars: gas pressure, electron degeneracy pressure and radiation pressure.

The gas pressure is:

$$P_g = \frac{N_A k}{\mu} \rho T \tag{11}$$

where μ is the mean molecular weight of the gas, which is related to X and Y via [6]:

$$\mu = \frac{2}{1 + 3X + 0.5Y} \tag{12}$$

Electron degeneracy can be obtained according to the following Equation [6]:

$$P_{e.d.} = \left(\frac{3}{8\pi}\right)^{2/3} \frac{h^2}{5m_e} \left(\frac{N_A \rho}{\mu_e}\right)^{5/3} \tag{13}$$

where μ_e is the mean molecular weight per electron:

$$\mu_e = \frac{2}{1 + X} \tag{14}$$

Finally, radiation pressure is obtained from:

$$P_{rad} = \frac{1}{3} a T^4 \tag{15}$$

Entropy

The entropy in the stellar interior can be calculated according to [6]:

$$S = const. + \frac{N_A k}{\mu} \left(\ln \frac{T^{3/2}}{\rho} + 4 \frac{P_{rad}}{P_g} \right) \tag{16}$$

Energy Generation Rate

Main sequence stars are hydrogen-burning. We consider both PP chain reactions (PP_I , PP_{II} and PP_{III}) and CNO bi-cycle. Reeves [19] and Novotny [15] have given expressions for energy generation rate. We use the convenient relations as given by Novotny [15]:

$$\epsilon_{PP} = \epsilon_{PP}(0) X^2 \rho f_{PP} T_7^m \tag{17}$$

$$\epsilon_{CNO} = \epsilon_{CNO}(0) X Z_{CN} \rho f_{CN} T_7^n \quad (\text{for } T_7 < 5) \tag{18}$$

$$\epsilon_{CNO} = \epsilon_{CNO}(0) X Z_{CN} \rho f_{CN} T_8^l \quad (\text{for } T_8 > 0.5) \tag{19}$$

where $T_b = T/10^b$ and m , n and l are temperature exponents $\left(\frac{d \ln \epsilon}{d \ln T}\right)_\rho$, and $\epsilon(0)$ is the rate of energy generation when there is no helium in the stellar core

($Y=0$). f_{PP} and f_{CN} are screening factors due to the presence of electrons and are given by the following Equations [15]:

$$f_{PP} = 1 + 0.188 \left[\frac{1}{2} (3 + X) \right]^{\frac{1}{2}} \rho^{\frac{1}{2}} T_6^{-\frac{3}{2}} \tag{20}$$

$$f_{CN} = 1 + 7 \times 0.188 \left[\frac{1}{2} (3 + X) \right]^{\frac{1}{2}} \rho^{\frac{1}{2}} T_6^{-\frac{3}{2}} \tag{21}$$

We see that in our chosen stellar mass range, the central temperature is always less than $T_8=5$. So, neglecting Equation (19), we combine (17), (18), (20) and (21) to get:

$$\begin{aligned} \epsilon = & \epsilon_{PP}(0) X^2 \rho \left[1 + 0.188 \left[\frac{1}{2} (3 + X) \right]^{\frac{1}{2}} \rho^{\frac{1}{2}} T_6^{-\frac{3}{2}} \right] T_7^m \\ & + \epsilon_{CNO}(0) X Z_{CN} \rho \left[1 + 1.316 \left[\frac{1}{2} (3 + X) \right]^{\frac{1}{2}} \rho^{\frac{1}{2}} T_6^{-\frac{3}{2}} \right] T_7^n \end{aligned} \tag{22}$$

Using the tables given by Novotny [15] for $\epsilon_{PP}(0)$, $\epsilon_{CNO}(0)$, m and n , we have derived convenient relations for them:

$$\epsilon_{PP}(0) = 0.084 T_7^{1.01} \tag{23}$$

$$\epsilon_{CNO}(0) = 3.35 \exp(1.95 T_7) \tag{24}$$

$$m = 4.93 T_7^{-0.389} \tag{25}$$

$$n = 22.9 T_7^{-0.346} \tag{26}$$

Opacity

Hereafter, we use the parameter Knudsen number which is defined as the ratio of the cosmion mean free path to the cosmion scale height:

$$Kn \equiv \frac{l_w}{r_w}$$

Since the cosmionic heat transfer is dominant in the core, the relevant opacity K must be found with due attention to the cosmion conduction. For two cases $Kn \gg 1$ and $Kn \ll 1$ the opacity is obtained according to following equations [17]:

$$\text{for } Kn \gg 1: \quad \kappa = \kappa_0 \left(\frac{T}{T_0}\right)^3 \left[1 + Kn_0^2 \left(\frac{T}{T_0} \frac{P_0}{P}\right)^2 \right] \tag{27}$$

$$\text{for } Kn \ll 1: \quad \kappa = \kappa_0 \left(\frac{T}{T_0}\right)^{5.5} \left(\frac{P}{P_0}\right)^{-1.5} \left(1 + Kn_0^2 \frac{P_0}{P}\right) \tag{28}$$

The subscript “0” refers to equilibrium values. For $Kn \sim 1$, T_w dose not change, but r_w varies as $\sim \rho^{-1/4}$ [17], so it can be shown that in this case the opacity will vary as $\kappa \sim T^3 T_w^{-1/2} \rho^{-3/4} (1 + Kn^2)$. Thus for $Kn \sim 1$, the following relation for κ can be found:

$$\kappa = \kappa_0 \left(\frac{T}{T_0} \right)^{15/4} \left(\frac{P}{P_0} \right)^{-3/4} \left[\frac{1 + Kn_0^2 \frac{P_0}{P}}{1 + Kn_0^2} \right] \quad (29)$$

Main Sequence Lifespan of Stars

Among several equations presented for main sequence lifespan by different authors we choose the following equation given by Clayton [6]:

$$T_{M.S.} = 12 \times 10^9 x \left(\frac{M}{M_\odot} \right)^{-3} \text{ years} \quad (30)$$

We assume that the age of the Galaxy to be $12 \times 10^9 x$ years.

POP II Stars

These stars came into existence at the time of galaxy formation. Since, according to Equation (30), the main sequence lifespan of those POP II stars, which are heavier than the sun, is less than the age of Galaxy, all these heavy stars have already left the main sequence and stars with masses roughly equal to the solar mass are just leaving the main sequence. Thus, we consider only those POP II stars which are less massive than the sun. So their age can be expressed simply as:

$$\tau_1 = T_G = 12 \times 10^9 X \text{ years} \quad (31)$$

POP I Stars

Hereafter we consider a typical POP I stars which has terminated one half of its main sequence lifespan or half the Galactic age, if the star is less massive than the sun. Thus we take:

$$\tau_2 = \frac{T_G}{2} = 6 \times 10^9 X \text{ years} \quad \left(\text{for } \frac{M}{M_\odot} < 1 \right) \quad (32)$$

$$\tau_3 = \frac{T_{M.S.}}{2} = 4.2 \times 10^9 \left(\frac{M}{M_\odot} \right)^{-3} \text{ years} \quad \left(\text{for } \frac{M}{M_\odot} \geq 1 \right) \quad (33)$$

Chemical Composition of the Stellar Cores

Hydrogen-burning causes the hydrogen mass fraction X to decrease in the core, the consumed H being transformed into He . Assuming that hydrogen-burning has been linear with time and neglecting elements heavier than helium, if at $t=0$ (the time of star

formation) X_c (the central mass fraction of H) is taken to have been the same as $X(0)$ (i.e. 0.70 for POP I and 0.76 for POP II), we get:

$$X_c(t) = X(0) \left[1 - \frac{t}{T_{M.S.}} \right] \quad (34)$$

Mean Molecular Weight at the Stellar Core

Using Equations (12), (14), (30) and (34) we obtain:

For POP I Stars with $\frac{M}{M_\odot} \geq 1$:

$$\mu = 0.85 \quad (35)$$

$$\mu_e = 1.48 \quad (36)$$

For POP I Stars with $\frac{M}{M_\odot} < 1$:

$$\mu = \frac{2}{3.24 - 0.963 \left(\frac{M}{M_\odot} \right)^3} \quad (37)$$

$$\mu_e = \frac{2}{1.7 - 0.385 \left(\frac{M}{M_\odot} \right)^3} \quad (38)$$

For POP II Stars with $\frac{M}{M_\odot} < 1$:

$$\mu = \frac{2}{3.4 - 1.9 \left(\frac{M}{M_\odot} \right)^3} \quad (39)$$

$$\mu_e = \frac{2}{1.76 - 0.76 \left(\frac{M}{M_\odot} \right)^3} \quad (40)$$

The Scope of Research

There are some factors and conditions which put constraints on the mass range we study.

In Relation with Temperature

The necessary temperature for starting hydrogen-burning is about $8 \times 10^6 \text{ }^\circ K$. Therefore, for both POP I and POP II stars, neglecting the effect of difference in chemical composition, the lower limit of our research is $\frac{M}{M_\odot} = 0.24$. The upper limit for population II stars is

$1 M_\odot$, because stars heavier than this have already left the main sequence. Moreover, we have inevitably

divided POP I stars into two groups with mass $\frac{M}{M_\odot} < 1$ and $\frac{M}{M_\odot} \geq 1$.

In this section, we will show that a convenient upper limit for the mass of POP I stars is $\frac{M}{M_\odot} \leq 20$.

Summing up, the mass range for POP II stars is $0.24 \leq \frac{M}{M_\odot} \leq 0.99$ while for POP I stars, the appropriate ranges are $0.24 \leq \frac{M}{M_\odot} < 1$ and $1 \leq \frac{M}{M_\odot} < 20$.

In Relation with Pressure

In general, low mass stars, with high central density and low temperature, have electron degeneracy pressure while in the case of heavy stars, which have a low density and high temperature, radiation pressure becomes considerable.

For POP I stars with $\frac{M}{M_\odot} \geq 0.19$ and POP II stars with $\frac{M}{M_\odot} \geq 0.20$, gas pressure is greater than electron degeneracy pressure, so that the latter can be neglected in the mass range $\frac{M}{M_\odot} \geq 0.24$.

Furthermore, if we compare gas pressure and radiation pressure, we see that $\beta \equiv \frac{P_g}{P_g + P_{rad}}$ is about 1 for stars with $\frac{M}{M_\odot} < 1$. As the mass increases, β will decrease and $1 - \beta \equiv \frac{P_g}{P_g + P_{rad}}$ will increase. For

example, in POP I stars with masses $\frac{M}{M_\odot} = 0.24, 0.99, 20$ and 100 , $1 - \beta$ is equal to $0.00003, 0.0006, 0.185$ and 0.851 , respectively.

As stated before, the upper limit in our calculations for stellar mass is $\frac{M}{M_\odot} = 20$. So by overlooking the radiation pressure we obtain a fairly good approximation.

In Relation with the Number of Captured Cosmions

The rate of transferred heat and temperature reduction through cosmionic thermal transport depend on the number of cosmions that have been captured by

the star during its lifetime.

According to Figure 3 in Gilliland *et al.* [9], cosmions with mass $m_w = 5m_p$, cross section $\sigma_w = 10^{-36} \text{ cm}^2$, and a relative number density (relative to number density of solar baryons) $\frac{n_w(0)}{n_b(0)} \cong 1.4 \times 10^{-11}$, can solve the solar neutrino problem. The number fraction of captured cosmions in a star is [14]:

$$n_{wb} = 5.6 \times 10^{-10} \left(\frac{t}{10^{17} \text{ s}} \right) \left(\frac{M}{M_\odot} \right)^{0.85} \times \left(\frac{m_p}{m_w} \right)^{-0.5} \min \left\{ 1, \frac{\sigma_s}{\sigma_c(M)} \right\} \tag{41}$$

Using this relationship and following our discussion about t , we have (for a $20 M_\odot$ star):

$$\left(\frac{n_{wb}}{n_{wb}} \right)_{star} = \left(\frac{n_{wb}}{n_{wb}} \right)_{sun} \left(\frac{M}{M_\odot} \right)^{-2.8} = 2.27 \times 10^{-5} \quad (z < 4)$$

$$\left(\frac{n_{wb}}{n_{wb}} \right)_{star} = \left(\frac{n_{wb}}{n_{wb}} \right)_{sun} \frac{z}{4} \left(\frac{M}{M_\odot} \right)^{-2.8} = 5.69 \times 10^{-6} z \quad (4 \leq z < 24)$$

$$\left(\frac{n_{wb}}{n_{wb}} \right)_{star} = \left(\frac{n_{wb}}{n_{wb}} \right)_{sun} \left(\frac{M}{M_\odot} \right)^{-2.2} = 1.37 \times 10^{-4} \quad (z \geq 24)$$

$$\tag{42}$$

in which z is a dimensionless number defined via $\sigma_s = z \times 10^{-36} \text{ cm}^2$.

The relative number of cosmions in stars heavier than $20 M_\odot$ is too low to be efficient in thermal transport and in producing any change in the stellar structure. Thus stars heavier than $20 M_\odot$ will be excluded from our research. In this case we will consider both *PP* chains and *CNO* bi-cycle in the energy generation rate and their parts in energy generation will be determined by $\epsilon_{pp}(0)$, $\epsilon_{cno}(0)$ and parameters m and n which are themselves functions of temperature.

Linear Perturbation Equations

Following Baker [3], Clayton [6] and Raby [17], we expand four parameters r, p, T and L about the time-independent equilibrium solutions to the first order in the infinitesimal dimensionless perturbations r', p', l' and t' :

$$r(M, t) = r_0(M) [1 + r'(M, t)] \tag{43}$$

$$p(M, t) = p_0(M) [1 + p'(M, t)] \tag{44}$$

$$T(M, t) = T_0(M) [1 + t'(M, t)] \tag{45}$$

$$L(M, t) = L_0(M) [1 + h'(M, t)] \tag{46}$$

Putting Equations (43-46) in the stellar structure Equations (1-4), we obtain the linear perturbation

equations. By properly combining the linearized equations, we then obtain a single third order, ordinary differential equation:

$$\frac{d^3 r'}{dt^3} + A \frac{d^2 r'}{dt^2} + B \frac{dr'}{dt} + Dr = 0 \tag{47}$$

where the parameters A , B and D and related parameters are:

$$A = \frac{3(-\kappa_P + 4 \frac{\alpha_0}{\delta_0} - \kappa_T \frac{\alpha_0}{\delta_0}) - \varepsilon_P - \varepsilon_T \frac{\alpha_0}{\delta_0}}{\frac{T_0}{\varepsilon_0} \nu_0 + \frac{T_0 \alpha_0}{\varepsilon_0 \delta_0} \mu_0} \tag{48}$$

$$B = \sigma_0^2 \left[\frac{-\nu_0(4 - \frac{3}{\alpha_0}) - \mu_0 \frac{\alpha_0}{\delta_0} (4 - \frac{3}{\alpha_0}) - \frac{3\nu_0}{\alpha_0}}{\nu_0 + \mu_0 \frac{\alpha_0}{\delta_0}} \right] \tag{49}$$

$$D = \sigma_0^2 \frac{\varepsilon_0}{T_0} \left(\frac{3(4 - \frac{3}{\alpha_0})(\kappa_P - 4 \frac{\alpha_0}{\delta_0} + \kappa_T \frac{\alpha_0}{\delta_0}) + (4 - \frac{3}{\alpha_0})(\varepsilon_P + \varepsilon_T \frac{\alpha_0}{\delta_0}) + 12 + 9 \frac{\kappa_P}{\alpha_0} + 3 \frac{\varepsilon_P}{\alpha_0}}{\frac{\alpha_0}{\delta_0} \mu_0 + \nu_0} \right) \tag{50}$$

$$\alpha_0 \equiv \frac{P_0}{\rho_0} \left(\frac{\partial \rho}{\partial P} \right)_T \equiv \left(\frac{\partial \ln \rho}{\partial \ln P} \right)_T \tag{51}$$

$$\delta_0 \equiv -\frac{T_0}{\rho_0} \left(\frac{\partial \rho}{\partial T} \right)_P \equiv -\left(\frac{\partial \ln \rho}{\partial \ln T} \right)_P \tag{52}$$

$$\sigma_0^2 \equiv \frac{GM}{r_0^3} = \left(\frac{4\pi G \rho_0}{3} \right) \tag{53}$$

$$\nu_0 \equiv P_0 \left(\frac{\partial S}{\partial T} \right)_T \tag{54}$$

$$\mu_0 \equiv T_0 \left(\frac{\partial S}{\partial T} \right)_P \tag{55}$$

$$\varepsilon_P \equiv \frac{P_0}{\varepsilon_0} \left(\frac{\partial \varepsilon}{\partial P} \right)_T \equiv \left(\frac{\partial \ln \varepsilon}{\partial \ln P} \right)_T \tag{56}$$

$$\varepsilon_T \equiv \frac{T_0}{\varepsilon_0} \left(\frac{\partial \varepsilon}{\partial T} \right)_P \equiv \left(\frac{\partial \ln \varepsilon}{\partial \ln T} \right)_P \tag{57}$$

$$\kappa_P \equiv \frac{P_0}{\kappa_0} \left(\frac{\partial \kappa}{\partial P} \right)_T \equiv \left(\frac{\partial \ln \kappa}{\partial \ln P} \right)_T \tag{58}$$

$$\kappa_T \equiv \frac{T_0}{\kappa_0} \left(\frac{\partial \kappa}{\partial T} \right)_P \equiv \left(\frac{\partial \ln \kappa}{\partial \ln T} \right)_P \tag{59}$$

Since Equation (47) is homogeneous in r' , we express its time-dependence as $r' = \xi e^{st}$.

Substituting r' and its derivatives in Equation (47) the following third order algebraic equation is obtained:

$$S^3 + AS^2 + BS + D = 0 \tag{60}$$

Introducing four parameters:

$$Q \equiv \frac{3B - A^2}{9} \tag{61}$$

$$R \equiv \frac{9AB - 27D - 2A^3}{54} \tag{62}$$

$$U \equiv \left(R + (Q^3 + R^2)^{1/2} \right)^{1/3} \tag{63}$$

$$W \equiv \left(R - (Q^3 + R^2)^{1/2} \right)^{1/3} \tag{64}$$

Solutions of Equation (60) are easily found:

$$S_0 = U + W - \frac{1}{3}A \tag{65}$$

$$S_{\pm} = -\frac{1}{2}(U + W) - \frac{1}{3}A \pm \frac{1}{2}i\sqrt{3}(U - W) \tag{66}$$

Obviously, instability occurs if $S_0 > 0$ or $R_e S_{\pm} > 0$.

Unstable Modes, Ignoring Radiation Pressure

For a perfect gas in which $P_{rad} = P_{e.d.} = 0$, we have (16 and 51-55):

$$\alpha_0 = 1 \tag{67}$$

$$\delta_0 = 1 \tag{68}$$

$$\nu_0 = -\frac{N_A k}{\mu} \tag{69}$$

$$\mu_0 = \frac{5 N_A k}{2 \mu} \tag{70}$$

Using Equations (27-29), (58) and (59) we get:

$$\kappa_T = 3 + \frac{2Kn_0^2}{1 + Kn_0^2} \quad Kn \gg 1 \tag{71}$$

$$\kappa_T = 5.5 \quad Kn \ll 1 \tag{72}$$

$$\kappa_T = \frac{15}{4} \quad Kn \sim 1 \tag{73}$$

$$\kappa_P = \frac{-2Kn_0^2}{1+Kn_0^2} \quad Kn \gg 1 \tag{74}$$

$$\kappa_P = -1.5 + \frac{2Kn_0^2}{1+Kn_0^2} \quad Kn \ll 1 \tag{75}$$

$$\kappa_P = \frac{-3}{4} - \frac{Kn_0^2}{1+Kn_0^2} \quad Kn \sim 1 \tag{76}$$

In order to calculate ε_T and ε_P we neglect with good approximation the derivatives of f_{PP} , f_{CN} , m , n , $\varepsilon_{PP}(0)$ and $\varepsilon_{CNO}(0)$ with respect to T . Thus, according to Equations (22), (56) and (57) we find:

$$\varepsilon_P = 1 \tag{77}$$

$$\varepsilon_T = m \frac{\varepsilon_{0,PP}}{\varepsilon_0} + n \frac{\varepsilon_{0,CNO}}{\varepsilon_0} - 1 \tag{78}$$

Note that

$$\varepsilon_0 = \varepsilon_{0,PP} + \varepsilon_{0,CNO} \tag{79}$$

Now A , B and D can be found:

$$A = \frac{3(4 - \kappa_P - \kappa_T) - \varepsilon_T - 1}{\frac{3 N_A k T_0}{2 \mu \varepsilon_0}} \tag{80}$$

$$B = \sigma_0^2 \tag{81}$$

$$D = \sigma_0^2 \left(\frac{12\kappa_P + 3\kappa_T + 4 + \varepsilon_T}{\frac{3 N_A k T_0}{2 \mu \varepsilon_0}} \right) \tag{82}$$

As $A \sim \frac{\mu \varepsilon_0}{T_7} \times 10^{-15}$, $B \sim 3\rho_C \times 10^{-7}$ and

$$D \sim \frac{3\mu \varepsilon_0}{T_7} \rho_C \times 10^{-22}$$

, among all the terms of U and W , only three terms AB , D and $B^{3/2}$ are considerable and the rest can be ignored:

$$U = \left(\frac{AB}{6} - \frac{D}{2} + \left(\frac{B^3}{27} \right)^{1/2} \right)^{1/3} \tag{83}$$

$$W = \left(\frac{AB}{6} - \frac{D}{2} - \left(\frac{B^3}{27} \right)^{1/2} \right)^{1/3} \tag{84}$$

Taking $\frac{AB}{6} - \frac{D}{2} = \delta$ and expanding, we get:

$$U+W = \left(\delta + \left(\frac{B^3}{27} \right)^{1/2} \right)^{1/3} + \left(\delta - \left(\frac{B^3}{27} \right)^{1/2} \right)^{1/3} \approx \frac{A}{3} - \frac{D}{B} \tag{85}$$

Thus, using Equations (65) and (66), we simply have:

$$S_0 \approx -\frac{D}{B} \tag{86}$$

$$S_{\pm} \approx \frac{D}{2B} - \frac{A}{2} \pm i\sqrt{B} \tag{87}$$

We are now ready to calculate the instability conditions for various values of Knudsen number, cosmion mass and cosmion cross section.

Here we must pay attention to Knudsen number-cosmion mass-cosmion cross section relations. A convenient relation has been given by Bouquet and Salati [5]:

$$Kn^{-1} \approx 3 \left(\frac{m_P}{m_W} \right)^{1.5} \left(\frac{\sigma_{cap}}{10^{-36} \text{ cm}^2} \right) \left(\frac{M}{M_{\odot}} \right)^{-0.2} \tag{88}$$

To study instability for different values of cosmion mass, we fix $\sigma_{cap}=4 \times 10^{-36} \text{ cm}^2$. Thus we have:

$$Kn^2 = \frac{1}{144} \left(\frac{m_W}{m_P} \right)^3 \left(\frac{M}{M_{\odot}} \right)^{0.4} \tag{89}$$

Similarly, we can choose a constant value $m_W=5m_P$ for cosmion mass and vary $\sigma_{cap}=4 \times 10^{-36} \text{ cm}^2$;

$$Kn^2 = \frac{125}{9} z^{-2} \left(\frac{M}{M_{\odot}} \right)^{0.4} \tag{90}$$

Using Equations (86) and (87) for instability conditions ($S_0 > 0$ and $ReS_{\pm} > 0$), Table 1 can be obtained for main sequence stars of difference masses. In this table:

$BI = \frac{\varepsilon_T + 13}{5 - \varepsilon_T}$	$BII = \frac{2\varepsilon_T + 11}{7 - 2\varepsilon_T}$
$BIII = \frac{\varepsilon_T + 2.5}{9.5 - \varepsilon_T}$	$BIV = \frac{2\varepsilon_T + 3.5}{11.5 - 2\varepsilon_T}$
$BV = \frac{4\varepsilon_T + 25}{32 - 4\varepsilon_T}$	$BVI = \frac{17 + 8\varepsilon_T}{43 - 8\varepsilon_T}$

Results and Discussion

Explicit results for all instability modes of main sequence stellar cores were obtained, distinguishing six conditions $Kn_0^2 - BI > 0$, $Kn_0^2 - BII > 0, \dots$ and $Kn_0^2 - BVI < 0$.

Table 1. Instability conditions for various values of Knudsen number, cosmion mass and cross section

Parameter varied	Instability	$\tau \sim \tau_{ff} \ll \tau_{rel}$ $\Leftrightarrow Kn \gg 1$	$\tau \sim \tau_{ff} \gg \tau_{rel}$ $\Leftrightarrow Kn \ll 1$	$\tau_{ff} \ll \tau < \tau_{rel}$ $\Leftrightarrow Kn \sim 1$
Kn	$S_0 > 0$	$Kn_0^2 - BI > 0$	$Kn_0^2 - BIII > 0$	$Kn_0^2 - BV > 0$
	$ReS_{\pm} > 0$	$Kn_0^2 - BII < 0$	$Kn_0^2 - BIV < 0$	$Kn_0^2 - BVI < 0$
m_W ($\sigma_{cap} = const.$)	$S_0 > 0$	$(\frac{m_m}{m_p})^3 - 144BI \times (\frac{M}{M_{\odot}})^{-4} > 0$	$(\frac{m_m}{m_p})^3 - 144BIII \times (\frac{M}{M_{\odot}})^{-4} > 0$	$(\frac{m_m}{m_p})^3 - 144BV \times (\frac{M}{M_{\odot}})^{-4} > 0$
	$ReS_{\pm} > 0$	$(\frac{m_m}{m_p})^3 - 144BII \times (\frac{M}{M_{\odot}})^{-4} < 0$	$(\frac{m_m}{m_p})^3 - 144BIV \times (\frac{M}{M_{\odot}})^{-4} < 0$	$(\frac{m_m}{m_p})^3 - 144BVI \times (\frac{M}{M_{\odot}})^{-4} < 0$
σ_{cap} ($m_W = const.$)	$S_0 > 0$	$Z^{-2} - \frac{9}{125}BI \times (\frac{M}{M_{\odot}})^{-4} > 0$	$Z^{-2} - \frac{9}{125}BIII \times (\frac{M}{M_{\odot}})^{-4} > 0$	$Z^{-2} - \frac{9}{125}BV \times (\frac{M}{M_{\odot}})^{-4} > 0$
	$ReS_{\pm} > 0$	$Z^{-2} - \frac{9}{125}BII \times (\frac{M}{M_{\odot}})^{-4} < 0$	$Z^{-2} - \frac{9}{125}BIV \times (\frac{M}{M_{\odot}})^{-4} < 0$	$Z^{-2} - \frac{9}{125}BVI \times (\frac{M}{M_{\odot}})^{-4} < 0$

We called the corresponding instability modes I, II, ... and VI, respectively. For the sake of brevity, we summarize the results as:

1. Instability modes I, IV, V and VI occur for population I stars in the mass range $0.24 \leq \frac{M}{M_{\odot}} \leq 5$ for various values of the cosmion mass and cross section.

2. The same instability modes occur for population II stars in the mass range $0.24 \leq \frac{M}{M_{\odot}} \leq 0.7$. Mode II occurs for population II stars heavier than $0.7 M_{\odot}$.

3. For heavy population I stars ($M > 5M_{\odot}$), instability modes I, II and V occur.

We conclude that cosmions with a mass of 4-10 GeV/c^2 and scattering cross section $0.2-11 \times 10^{-36} \text{ cm}^2$, previously suggested to constitute the Galactic halo and to solve both dark matter and solar neutrino problems, may cause thermal instabilities in the core of the sun and many other main sequence stars which have captured an efficient number of cosmions.

It should be stressed that the analysis followed in this paper was a local one and only linear perturbations were taken into account. A more exact investigation must consider both a global analysis and nonlinear dynamical effects.

Such a wide range of instability modes, if present weaken the cosmion hypothesis in the mass and cross section region which is interesting in the context of the dark matter and solar neutrino problems.

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