**Journal of Sciences, Islamic Republic of Iran 20(1): 73-78 (2009)** http://jsciences.ut.ac.ir
University of Tehran, ISSN 1016-1104

# Some New Properties of the Searching Probability Used in Design Comparison

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Received: 5 February 2008 / Revised: 23 December 2008 / Accepted: 17 January 2009

## Abstract

Consider search designs for searching one nonzero 2- or 3-factor interaction under the search linear model. In the noisy case, search probability is given by Shirakura et al. (Ann. Statist. 24(6) (1996) 2560). In this paper some new properties of the searching probability are presented. New properties of the search probability enable us to compare designs, which depend on an unknown parameter $\rho$, for all values of the parameter without needing to check for different values of $\rho$. Two new quantitative interval scale and parameter free criteria based on search probabilities are proposed for design comparison. These criteria are used to compare given designs. The equivalent search designs is defined based on new proposed criteria and present a class of equivalent designs which are orthogonal arrays of strength 2.

**Keywords:** Equivalent designs; Interactions; Main effect plans; Search designs; Search models

## Introduction

Consider a $2^m$ factorial experiment with $n$ runs, where $n < 2^m$. A main effects plan allows us to make inference on the general mean and main effects under the assumption that 2-factor and higher order interactions are all zero. Such an assumption may not be valid in reality. There may be a few non-negligible interactions present. Consequently, the estimates of the parameters are biased. Search linear models introduced in [7] permit the search for non-negligible interactions from 2-factor and higher order interactions. Now, we consider the following search linear model for a $2^m$ factorial experiment:

$$E\left(\underline{y}\right) = A_1\xi_1 + A_2\xi_2, \quad \mathrm{var}\left(\underline{y}\right) = \sigma^2 I \qquad (1)$$

where $\underline{y}(n\times 1)$ is a vector of observations, $\xi_1(\nu_1\times 1)$, $\nu_1 = 1+m$, is the vector of the general mean and main effects, and $\xi_2(\nu_2\times 1)$, $\nu_2 = \binom{m}{2}+\binom{m}{3}$, is the vector of 2- and 3-factor interactions, $A_1$ and $A_2$ are design matrices, and $\sigma^2$ is an unknown nonnegative constant. The problem is to search for the one nonzero element of $\xi_2$ and draw inference on it in addition to the elements of $\xi_1$. The designs that resolve this problem are called main effect plus one (MEP.1) plans. When $\xi_2 = 0$, the model is the main effects model. Based on the basic mathematical formulation of the problem introduced by Srivastava [7], the following theorem is fundamental for the given search linear model.

*MSC*: primary 62K15

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**Theorem 1.1.** Consider the model (1) and let $\sigma^2 = 0$. A necessary and sufficient condition that the search and inference problem can be completely solved is that for every submatrix $A_{22}(n \times 2)$ of $A_2$,

$$rank\left(A_1 : A_{22}\right) = \nu_1 + 2 \quad (2)$$

holds.

Such a design is called a search design. By this theorem, one considers a class of linear models containing the general mean, main effects, and one interaction. There are $\nu_2$ models in this class, say M. Each has the general mean and main effects as common parameters, but they are different in one possible interaction; i.e.

$$\mathrm{M} = \left\{ \mathrm{M}(\zeta) : E\left(\underline{y}\right) = A_1\xi_1 + a(\zeta)\zeta, \quad \zeta \in \xi_2 \right\} \quad (3)$$

where $a(\zeta)$ is the $n \times 1$ column of $A_2$ corresponding to $\zeta$. The case when $\sigma^2$=0 is called the noiseless case. However, in practical experiments we have the noisy case $\sigma^2 > 0$ in which the condition (2) is still necessary. For the noisy case, Srivastava [7] proposed a procedure for solving the search problem, which correspond to minimization of the sum of squares due to error (SSE) of $\nu_2$ candidate models in class M. That is, consider models, say $M_1$ and $M_2$, from class M; one says $M_1$ provides a better fit and is selected over $M_2$ if SSE($M_1$) < SSE($M_2$). For the noiseless case, this is done with probability one. However, this is not assured for the noisy case and, for a given search design, the stochastic properties of the SSE have to be considered. In this case, the probability

$$P\left(SSE\left(M\left(\zeta_0\right)\right) < SSE\left(M\left(\zeta\right)\right) \middle| \zeta_0, \zeta, \sigma^2\right), \quad (4)$$

where $M(\zeta_0)$ is the possible true model and $M(\zeta)$ for $\zeta(\neq \zeta_0)$ is any other model in M, is not necessarily equal to 1 and, possibly, is less than 1. The value of the probability depends on the value of $\sigma^2$. This probability, called the "*searching probability*" ($SP$), also depends on the search design. So, one may be interested in the value of the SP for a given design, using that for design comparison. Based on SPs, several criteria have been developed for comparing search designs, and all depend on the value of the unknown actual quantity of the nonzero interaction effect, $\rho = |\zeta_0| / \sigma$, where $\zeta_0$ is the possible nonzero effect of $\xi_2$. Shirakura et al. [6] compared search designs for different reasonable possible values of $\rho$. Ghosh and Teschmacher[3] gave modified criteria to perform the comparison once without going through different values of $\rho$. However, using these criteria, the information is sacrificed due to simplification and reduction of the enormous task of comparison for different values of $\rho$.

In this paper, the problem of the dependence of the search probability to $\rho$, which causes an enormous task or losing information due to simplification for comparing designs, is tackled. To overcome this problem some new properties of searching probabilities are derived and presented to help in simplification and reduction of the number of comparisons task without losing information in design selection.

## Materials and Methods

### *Notation and Preliminary Results*

In this section, notation and previous results are presented. They are, mostly, adapted from material given in [6], and are used in subsequent sections of this work. The notation is

$Q = A_1(A'A_1)^{-1}A_1'$ is an $n \times n$ matrix of rank $\nu_1(< n)$,

$r(\zeta) = a(\zeta)'(I - Q)a(\zeta)$ is a positive constant that depends on the search design,

$b(\zeta) = (I - Q)a(\zeta) / \sqrt{r(\zeta)}$ is an $n \times 1$ vector, for each $\zeta \in \xi_2$,

$x = b(\zeta)'b(\zeta_0)$ is a constant with $-1 < x < 1$,

$\rho = |\zeta_0| / \sigma$ is the absolute value of the actual unknown effect,

$c_1 = \sqrt{\frac{r(\zeta_0)}{2}(1-x)}$, $c_2 = \sqrt{\frac{r(\zeta_0)}{2}(1+x)}$ are positive constants where for $0 \le x \le 1$ it is true that $c_1 \le c_2$, $0 \le c_1 \le \sqrt{r(\zeta_0)/2}$, and $\sqrt{r(\zeta_0)/2} \le c_2 \le \sqrt{r(\zeta_0)}$.

Assuming $\underline{y}$ has a normal distribution with its mean given as the mean of the true model $M(\zeta_0)$ in M and variance $\sigma^2 I$, then the searching probability of (4) can be written as the following expression

$$G(x, \rho) = 1 - \Phi(c_1\rho) - \Phi(c_2\rho) + 2\Phi(c_1\rho)\Phi(c_2\rho) \quad (5)$$

where $\Phi(.)$ is the Standard Normal pdf.

**Theorem 2.1.** [6] The function $G(x,\rho)$ has the following properties:

($i$) $G(-x,\rho)=G(x,\rho)$.

($ii$) $0.5\le G(x,\rho)\le 1$.

($iii$) For a fixed $x$, $G(x,\rho)$ is continuous and strictly increasing in $\rho$.

($iv$) For a fixed $r(\zeta_0)$ and $\rho$, $G(x,\rho)$ is monotonically increased as $c_1$ is increased.

Although the actual value of the effect $\rho$ is restricted to $|\zeta_0|$ and hence positive values but for negative $\zeta_0$ one can show that $G(x,\rho)=G(x,-\rho)$. Therefore, by Theorem 2.1($i$), without loss of generality we consider nonnegative $\rho=|\zeta_0|/\sigma$ and $0<x<1$. Note also from Theorem 2.1($iii$) that, as $\rho$ tends to $\infty$, the probability function $G$ increases to $1$.

## Results

### *Some New Properties for Search Probabilities*

The search probability in (5) depends on the given search design $T$ through $c_1$ and $c_2$. It also depends on the actual value of the effect $\rho$. So, the SP can be written as $P_T(\rho)$. The following lemma address a property of $P_T(\rho)$.

**Lemma 3.1.** For a given search design $T$, and fixed $c_1$, there exists a positive point $w$ depending on $T$, such that $P_T(\rho)$ is convex in $0\le\rho\le w$ and concave for $\rho>w$.

**proof.** For $P_T(\rho)$ given in (5) we have

$$\frac{\partial^2}{\partial\rho^2}P_T(\rho)=\frac{1}{\pi}e^{-\frac{r(\zeta_0)}{2}\rho^2}(A-B) \qquad (6)$$

Such that

$$B=\sqrt{\frac{\pi}{2}}\rho\Big[c_1^3\left(2\Phi(c_2\rho)-1\right)e^{\frac{1}{2}c_2^2\rho^2} +c_2^3\left(2\Phi(c_1\rho)-1\right)e^{\frac{1}{2}c_1^2\rho^2}\Big]$$

and $A=2c_1c_2$ are both positive, and $\frac{1}{\pi}e^{-\frac{r(\zeta_0)}{2}\rho^2}$ is also positive. Note that $A$ is a constant and $B$ is an increasing function of $\rho>0$. So, for $\rho=0$ we have $A>0$, $B=0$ and then $A-B>0$. As $\rho$ increases, $A-B$ tends to zero. Let $\rho=w>0$ be the point at which $A-B=0$. Clearly, for $\rho>w$ we have $A-B<0$. Therefore, by Theorem 2.1(iii), the proof is complete.

The point $w$ is the inflection point and its value depends on the design $T$ through $r(\zeta_0)$. For $b>w$ the point $w$ partitions this interval into two parts, $[0,w)$ and $[w,b]$ for $\rho$. For a search design with a high SP value the portion $[0,w)$ is small since both $A$ and $B$ increase as $r(\zeta_0)$ increases. However, this rate in $B$ is sharper than $A$. It verifies that, for a design with large $r(\zeta_0)$, $A-B$ in Lemma 3.1 switches from positive to negative at a low value of $\rho$. This is also true for moderate $r(\zeta_0)$ and large to moderate $c_1$. These lead us to assume that $P_T(\rho)$ is concave for all $\rho$ greater than a reasonable low value of $\rho$. This assumption might fail for a design with small $r(\zeta_0)$ and $c_1$, which is not of interest due to lack of qualification for comparison. Note that a nonzero effect $\zeta_0$ will generally be nonsignificant if $\rho<1$. Consequently, our subsequent work will restrict attention to values of $\rho\ge 1$. Under this restriction and by Lemma 3.1, the following results are established.

**Corollary 3.2.** For a given design $T$, let $P_T(\rho)$ be a concave function on $[a,b]$. The following properties are true:

$$(i)\ \frac{P_T(\rho)-P_T(a)}{\rho-a}\ge\frac{P_T(b)-P_T(a)}{b-a},\ \text{for } \rho\in[a,b] \qquad (7)$$

($ii$) For $a\le\rho_1\le\rho_2\le b$,

$$\frac{P_T(\rho_1)-P_T(a)}{\rho_1-a}\ge\frac{P_T(b)-P_T(\rho_2)}{b-\rho_2} \qquad (8)$$

**proof.** This is clear from Lemma 3.1.

By properties (i) and (ii) in corollary 3.2, it is clear that, as $\rho$ increases, the slope of the curve $P_T(\rho)$ decreases. That is, the rate of increase in values of $P_T(\rho)$ will get slower as $\rho$ increases. This means that lower values of $\rho$ have a greater effect on the area under the curve $P_T(\rho)$, denoted by $A_T$, than the higher values of $\rho$. With this in mind, the following proposition is presented.

**Proposition 3.3.** Consider $A_T=\int_a^b P_T(\rho)d\rho$ for the concave curve $P_T(\rho)$ on $[a,b]$. Let define the area $S_T$ by $S_T = \frac{1}{2}[b-a]\left[P_T(a)+P_T(b)\right]$. Then $A_T \geq S_T$.

**proof.** For any $\rho \in [a,b]$, there exists an $0 \leq \alpha \leq 1$ such that $\rho$ can be written as $\rho = \alpha a + (1-\alpha)b$. The proof is straight forward as $P_T(\rho) \geq \alpha P_T(a) + (1-\alpha)P_T(b)$ from Lemma 3.1.

Clearly, for a given design $T$ as $P_T(\rho)$ get larger it is more likely that design $T$ identifies the true model from the incorrect one. It is also clear that the value of $S_T$ of a design with higher SP is higher than that of a design with lower SP. With this in mind we suggest approximate $A_T$ by $S_T$ for to simplify design comparison for $\rho \in [a,b]$.

For $a=1$ and $b \geq 3$, and given search designs $T_1$ and $T_2$, by Theorem 2.1 (i) and (iii) for sufficiently large $\rho$'s there exists a small $\varepsilon$ such that

$$b = \inf\left\{\rho : \left|P_{T_1}(\rho) - P_{T_2}(\rho)\right| < \varepsilon\right\}. \tag{9}$$

Therefore, for such an $a$ and $b$, the following theorem is established.

**Theorem 3.4.** Consider the associated search probabilities $P_{T_i}(\rho)$ for given designs $T_1$ and $T_2$. If $P_{T_1}(\rho) > P_{T_2}(\rho)$ at $\rho = 1$, then $S_{T_1} \geq S_{T_2}$.

**proof.** By $b$ as defined in (9), $P_{T_1}(\rho) \cong P_{T_2}(\rho)$. Then the proof is clear from proposition 3.3 by substituting $S_{T_i} = [b-1][P_{T_i}(1)]/2$.

In the next section, these results will be used in comparing search designs.

### *Design Comparison Criteria*

To compare search designs for a given values of $\rho$, the minimum searching probability criterion is suggested in [6]. Ghosh and Teschmacher [3] proposed a new type of ordinal criteria and generalized it so that a comparison of search designs can be made without requiring a specific value of $\rho$. However, this is at the cost of losing some information, which may cause difficulty in decision making. The results of the previous section are applicable to these criteria for comparing designs with more possible information when one just uses search probabilities for $\rho = 1$.

In this paper, new criteria are proposed that consider the quantity of paired searching probability differences. It is reasonable since, in comparing designs, one may come up with a situation in which there is a majority with positive differences in favor of a design, yet with small values compared to the minority of negative differences with large absolute values.

For a design $T_i$, let $\text{SPM}_i$ be the $\nu_2 \times \nu_2$ matrix of searching probabilities for $\rho = 1$. That is, the columns of $\text{SPM}_i$ correspond to all $\zeta_0$ representing the possible true nonzero effect and the rows correspond to all $\zeta$ representing other effects in $\xi_2$. Note that, for a given $\zeta_0 \in \xi_2$, the elements in the corresponding column are the searching probabilities, each of which represents how likely it is that the true effect $\zeta_0$ is distinguishable from another effect $\zeta(\neq \zeta_0) \in \xi_2$. Averaging the SPs in a given column over $\zeta(\neq \zeta_0)$ is the distinguishable mean search probability of $\zeta_0$. Since $\zeta \neq \zeta_0$, the diagonal elements of SPM are not of interest.

### *Global Mean Criterion*

For two search designs, $T_i$ and $T_j$, define

$$g_{ij} = \frac{1}{\nu_2(\nu_2 - 1)} tr\left[(\mathbf{J}-\mathbf{I})\left(\mathbf{SPM}_i - \mathbf{SPM}_j\right)\right] \tag{10}$$

where $\mathbf{J}$ is a $\nu_2 \times \nu_2$ matrix of ones and $\mathbf{I}$ is the identity matrix of order $\nu_2$. Consequently, $T_i$ is said to be better than $T_j$ if $g_{ij} > 0$. We call this the Global Mean criterion and denote it by G-criterion.

### *Effectwise Mean Criterion*

In the class of models considered in section 1 it is known that only one of the effects, say $\zeta_0$, is the true nonzero interaction effect in $\xi_2$. So, we focus on each of the columns in SPM. Now, for given designs $T_i$ and $T_j$, we consider the $\nu_2 \times \nu_2$ diagonal matrix $D_{ij}$

$$D_{ij} = diag\left[\frac{1}{\nu_2 - 1}(\mathbf{J}-\mathbf{I})\left(\mathbf{SPM}_i - \mathbf{SPM}_j\right)\right] = (d_{tt}). \tag{11}$$

A $d_{tt} > 0$ means that the design $T_i$ distinguishes the t-associated nonzero $\zeta_0 \in \xi_2$ with a higher average search probability than does $T_j$. Now, let $d_{ij}^{+}(d_{ij}^{-})$ be the number of positive (negative) diagonal elements $d_{tt}$, $t = 1,2,...,\nu_2$. Let $d_{ij}^{0}$ be the number of $d_{tt}$ with value zero. Clearly, $0 \leq d_{ij}^{+} \leq \nu_2$, and $d_{ij}^{+} + d_{ij}^{-} + d_{ij}^{0} = \nu_2$. The following rules discriminate designs.

1. If $d_{ij}^{+} = \nu_2$, then $T_i$ is *efficiently* better than $T_j$. If $d_{ij}^{-} = \nu_2$, $T_j$ is *efficiently* better than $T_i$.

2. If the majority of $d_{tt}$'s are positive(negative); i.e. $d_{ij}^{+} > \nu_2/2$ ($d_{ij}^{-} > \nu_2/2$), then $T_i$ ($T_j$) is considered *relatively* better than $T_j$ ($T_i$).

In the case that ties exist (and hence zero diagonal elements) if neither the positive nor negative number of diagonal elements achieve a majority, then under the condition $d_{ij}^{+} = d_{ij}^{-} > 0$, the comparison between $T_i$ and $T_j$ might be inconclusive. We call this the Effectwise Mean criterion and denote it by E-criterion.

***Isomorphic Designs***

It can happen that two or more search designs have the same searching probability efficiency in searching for a nonzero interaction effect with respect to the criterion which is applied to them for comparison purposes.

**Definition 4.1.** Let $T_i$ and $T_j$ be two search designs with $n$ runs. They said to be equivalent or isomorphic in the G-criterion (are *G-isomorphic*) if $g_{ij} = 0$. They are also said to be equivalent in the E-criterion (are *E-isomorphic*) if $d_{ij}^{0} = \nu_2$. Obviously, if designs are E-isomorphic, then they are G-isomorphic, but the converse is not true.

***Implementation***

The utility of the G- and E-criteria defined above in design comparison will be presented in this section. First, the criteria are applied to MEP.1 plans for $m = 7$. The following design $T_1$ given in [2] is a MEP.1 plan with $n = 15$ runs. It is a balanced array of strength 2, composed of $T_{11}$ and $T_{12}$, i.e. $T_1' = [T_{11}' : T_{12}']$ where for the vector of ones $J_{7\times1}$, $T_{11}' = [J_{7\times1} : I_7]$ and $T_{22}'$ is the incidence matrix of a symmetric BIB design with parameters $b = v = 7$ and $r = k = 4$.

$$T_1' = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

Let us take the MEP.1 plan for $m = 7$ with $n = 15$ runs given in [5] as $T_2$. Using the G-criteria for comparing $T_1$ and $T_2$ gives $g_{12} = 0.033154 > 0$. Therefore, $T_1$ is better than $T_2$. Now, use the E-criterion to determine $d_{12}^{+}$. It gives $d_{12}^{+} = 56$ which means $T_1$ is efficiently better than $T_2$.

Next, we consider the case of $m = 4$ with $n = 12$ runs. For this case designs are compared in two parts according to the designs which are selected for comparison.

*Part 1.* In this part we repeat the comparison of the designs $D_1$, $D_2$ and $D_3$ given in [3] on the basis of their criterion I. However, we enjoy the advantage of Theorem 3.4 in decision making. The G- and E-criteria is also considered in comparing these designs. The results are presented in Table 1. All of the criteria confirm that $D_1$ is better than $D_2$ and $D_3$, and, in fact, it is better than $D_2$ efficiently by E-criterion. Using Theorem 3.4 in comparing $D_2$ and $D_3$, the last row of Table 1 shows that $D_2$ is better than $D_3$ under criterion I. The G-criterion and E-criterion give the same result, though $D_2$ is relatively better than $D_3$.

*Part 2.* In this part, consider the class of plans for $m = 4$ obtained as follow from $T_1'$ in (12). Consider $T_{11}' = [J_{4\times1} : I_4]$ and, for $m = 4$, let $T_{12}$ be obtained by deleting any three of the seven columns from $T_{12}$ of $m = 7$ in (12). Ghosh and Talebi [2] showed that the resulting $T' = [T_{11}' : T_{12}']$ are isomorphic 12-run plans which are MEP.1 plans for $m = 4$. We compute the SPM for all 35 designs in this class. These SPMs are equivalent in having two distinct elements $f = 0.967265$ and $h = 0.944356$ as off-diagonal elements. The values $f$ and $h$ occurs 6 and 3 times respectively in each column of SPM. For any pair of designs, say $T_i$ and $T_j$, $1 \le i < j \le 35$, in this class the $g_{ij}$ of G-criterion and $d_{ij}^{\cdot}$ s of E-criterion are determined. The result is $g_{ij} = 0$, $d_{ij}^{+} = d_{ij}^{-} = 0$, and $d_{ij}^{0} = 10$, for $1 \le i < j \le 35$. It means, by definition 4.1, that all of these MEP.1 plans are E-isomorphic and

**Table 1.**

| Comparison | Criterion I | | | G-Criterion | E-Criterion | | |
|---|---|---|---|---|---|---|---|
| | $n_{ij}^{+}$ | $n_{ij}^{-}$ | $n_{ij}^{0}$ | $g_{ij}$ | $d_{ij}^{+}$ | $d_{ij}^{-}$ | $d_{ij}^{0}$ |
| $D_1$ *vs* $D_2$ | 78 | 0 | 12 | 0.01170 | 10 | 0 | 0 |
| $D_1$ *vs* $D_3$ | 73 | 17 | 0 | 0.01661 | 9 | 1 | 0 |
| $D_2$ *vs* $D_3$ | 47 | 43 | 0 | 0.00491 | 7 | 3 | 0 |

hence G-isomorphic search designs. Note also that the average searching probability of the columns are equal, which means that the designs are invariant in distinguishing any one of the interaction effect as the possible nonzero $\zeta_0 \in \xi_2$.

## Discussion

For a $2^m$ factorial experiment, search designs are considered for searching and making inference on one interaction from 2- or 3-factor interactions under the search linear model. The condition given in (2) is necessary and sufficient for searching and estimating the problem in the noiseless case. However, it is only a necessary condition in the noisy case. Therefore, the search designs obtained in the noiseless case may not identify the true interaction and discriminate the correct model perfectly for the noisy case. In this case, the search probability given in (4) to discriminate between the correct and incorrect models is less than one, so it is most desirable to have a design with as high a search probability as possible. By now, it has been known that most of the search designs obtained for the noiseless case, for example, [1], [4] and [3], are all desirable for the noisy case with a relatively high SP. The positive value $w$ proposed, as the inflection point in Lemma 3.1 is guaranteed to be less than 1 as long as $r(\zeta_0)$ is greater than 2, which is true for almost all of these designs. It means that the search probability $P_T(\rho)$ for comparable search designs is a concave function for $\rho > w$ with $w < 1$. That is, $P_T(\rho)$ has a sharp slope for $w \le \rho \le 1$ which causes it to tend to a value close to 1 rapidly. Under this situation, comparing the designs for lower values of $\rho$, at which the significant nonzero interaction effect is small compared to probably a large noise, is more difficult to be identified yet, important than at higher values of $\rho$ where $P_T(\rho)$ is flat. Under this scenario, the properties of the searching probability, which are obtained, in this paper are quite helpful in design comparison. Keeping this in mind, the G- and E-criteria based on search probabilities are presented and applied to candidate designs to determine the design which is most likely to identify the nonzero interaction and discriminate the correct model. In addition to these criteria, it was shown that, by using the new properties of the SP, criterion I given in [3] is not inconclusive in determining the better design between $D_2$ and $D_3$ any more. G- and E-isomorphic(equivalent) search designs are defined and a class of the equivalent search designs, all are of orthogonal arrays of strength 2, is presented for $m = 4$.

## Acknowledgment

I thank Dr. Ken G. Russell, Associate Professor of Statistics at School of Mathematics and Applied Statistics, UoW, Australia, for his careful reading of this manuscript and two referees for their valuable comments. This research is supported by Research and Technology Affair of the University of Isfahan grant 860820.

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