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Short Communication

LINEAR PULSATION CHARACTERISTICS OF MIRA VARIABLE STARS

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Abstract

The linear adiabatic pulsation-periods of Mira variable stars have been derived. Approximately 270[1] *models were calculated for* $M = 0.7\ M_\Theta$ *to* $2\ M_\Theta$ *stars with radii from* $180\ R_\Theta$ *to* $340\ R_\Theta$ *and luminosities from* $2800\ L_\Theta$ *to* $10{,}000\ L_\Theta$*. The chemical composition of all models is* $(X,Z) = (0.7,0.02)$*. From the result of this study, linear relations on Luminosity-Period-Mass relationship and luminosity-period-radius-mass relationship have been derived for the fundamental mode. The relation has been tested on a recent data set of Mira stars in the LMC.*

Keywords: *L.M.C.; Mass-luminosity relation; Mira stars; Stellar pulsation*

1. Introduction

The pulsation of the RR Lyrae stars and classical Cepheids has been well studied and relatively well understood phenomenon both theoretical and observationally. In contrast, the pulsation of Mira variable stars is still the subject of controversial investigation. One of the reasons why the theoretical study of Mira stars is difficult is partly based on the complexity of atmospheric structures. Another reason has come from the lack of well-determined period-luminosity relation based on observational data.

Fortunately, after the effort for several years, the observational period-luminosity relationship was established ([3,4], hereinafter FGWC). Ostlie and Cox [8] performed a careful investigation on the theoretical periods of Mira stars, but didn't succeed in comparing their results with the observed period-luminosity relation. It seems useful to compare the theatrical and observational period-luminosity relation even if the linear adiabatic periods are used. The main aim of this study was to show the linear adiabatic periods of Mira stars and to compare them the results of FGWC. The mass luminosity relation is derived for the LMC Mira stars [5,7,13].

2. Models and Numerical Results

In the present study, we calculate the models of Mira stars for the range of mass from $0.7\ M_\Theta$ *to* $2\ M_\Theta$ *with radii from* $180\ R_\Theta$ *to* $340\ R_\Theta$ *and luminosities from* $2800\ L_\Theta$ *to* $10{,}000\ L_\Theta$*. The chemical composition of all models is* $(X,Z) = (0.7,0.02)$ *[1,13]. The programming code is essentially identical with that of Takeuti [12] for calculating linear adiabatic radial pulsation of super giant stars. The convective energy transport is included in the code. The ratio of mixing-length to the pressure scale height, L/H, is fixed as one through the models. The relation derived from Bohm-Vitnese [2] is used for the temperature distribution of atmosphere. The most outer shell of atmospheres is determined by the iteration including the effect of sphericity. The opacity formula of Stellingwerf [10,11] also used without any special Modification for the effect of molecules. The procedures*

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[1] *Models of various computational input are available through the author.*

seem too simple compared with that in Ostlie and Cox [8]. The difference with the present is the opacities, the atmospheric structure, the inner boundary, and the mixing-length of convection. The present models are solved as a marching problem although Ostlie and Cox [8] used Caster's method. The present code chooses the length of every step automatically for any given accuracy. The code stops the integration if the temperature exceeds 10^7 K.

The effect of mass limit at the bottom of model envelops is carefully studied. We choose 0.5 stellar mass since the periods converge effectively into a definite value with the accuracy of less than 0.5 day. Even through the careful analysis of Ostlie and Cox [8] should give very accurate periods, the present result may be useful to be compared with observation.

By using a least-square fitting program the luminosity-periods-mass (LPM) relation is derived as the following.

$$\begin{array}{llll} \mathrm{Log}\,(L/L_\Theta) = & & & \\ 0.5406\,\mathrm{Log}\,P\ + & 1.1010\,\mathrm{Log}\,(M/M_\Theta) + & 2.171 & \\ \pm 0.0083 & \pm 0.0150 & \pm 0.023 & (1) \\ & \sigma = 0.0273 & & \end{array}$$

where L is the luminosity, P, the period in days, and M, the mass, respectively. σ is the standard deviation. The luminosity-period-mass-radius (LPMR) relation is also derived as the following.

$$\begin{array}{lll} \mathrm{Log}\,(L/L_\Theta) = 0.4715\,\mathrm{Log}\,P & & \\ \quad \pm 0.01905 & & \\ \quad + 0.2403\,\mathrm{Log}\,(R/R_\Theta) & & \\ \quad \pm 0.598 & & \\ \quad + 0.9610\,\mathrm{Log}\,(M/M_\Theta) + 1.8032 & & (2) \\ \quad \pm 0.0381 \qquad \pm 0.0944 & & \\ \sigma = 0.0265 & & \end{array}$$

where R is the radius.

For further discussion, the relation among the luminosity, the periods, the mass, and the effective temperature, T, from Equation (2).

$$\mathrm{Log}\,(L/L_\Theta) = 0.563\,\mathrm{Log}\,P + 1.092\,\mathrm{Log}\,(M/M_\Theta) - 0.546\,\mathrm{Log}\,T + 4.104. \qquad (3)$$

3. Comparison with Observation

In Figure 1, nine lines of constant masses have been plotted in a Log L-Log P plane. The lines are derived from Equation (1) for masses between 0.6 M_Θ and 2.2 M_Θ with the interval of 0.2 M_Θ Also included in the diagram is observational data of Mira stars in the LMC (FGWC). It is shown that the masses of Mira stars seem to increase with the increase of both the period and luminosity. From Figure 1, we can estimate mass and luminosity of Mira stars such as M = 1.4 (±0.2) 0.6 M_Θ and luminosity above 5000 L_Θ In this case the adiabatic and nonadiabatic fundamental mode periods shows small difference [8, Table 2]. On the other hand they determined the mixing length parameter by fitting the model to the observed luminosity and temperature, but this could be a correct procedure if the effective temperature was well as the star total mass was known to a much better accuracy. So, we have assumed the same mixing length value for all stars (see also [9]).

Also, in Figure 2, four lines of constant radius have been plotted in the Log L-Log P plane. The lines are derived from Equation (2) for radii between 250 R_Θ to 400 R_Θ with the interval of 50 R_Θ for fixed masses with M = 0.6 M_Θ, 1.2 M_Θ and 2.2 M_Θ. In Figure 3, three lines of constant temperature for T = 3000 K, 3500 K and 4000 K are shown in the same plane from Equation (3) for fixed masses with M = 0.6 M_Θ, 1.2 M_Θ and 2.2 M_Θ. The observational data points are included in both diagrams. It is interesting that the difference of effective temperatures of the models affects very slightly the periods and masses. The changes of effective temperature from 4000 K to 3000 K, which make the radii increase approximately 40%, produce the decrease of masses by 12% for a fixed period.

For a further discussion from Equation (1), the masses of each observational data are calculated and linear relation between the luminosity and the masses has been derived as the following.

$$\begin{array}{lll} \mathrm{Log}\,(L/L_\Theta) = 1.654\,\mathrm{Log}\,(M/M_\Theta) + 3.390, & & \\ \qquad \pm 0.105 \qquad\qquad \pm 0.015 & & (4) \\ \qquad \sigma = 0.0573. & & \end{array}$$

The result is plotted in Figure (4), to show the dependence of luminosity on the mass.

It is supposed that Miras are on the asymptotic giant branch (GAB). If so, the core mass of these stars should be related by a simple relation to the luminosity, i.e.

$$L/L_\Theta) = 6.34.10^4\,(Mc/M_\Theta - 0.44)\,(M/7M_\Theta)^{0.19}, \qquad (5)$$

By Iben and Truran [6], where Mc is the core mass. Replace L in Equation (5) by that in Equation (4). Then we have

$$\mathrm{Log}\,(Mc/M_\Theta - 0.44) = 1.464\,\mathrm{Log}\,(Mc/M_\Theta) - 1.251.$$

(6)

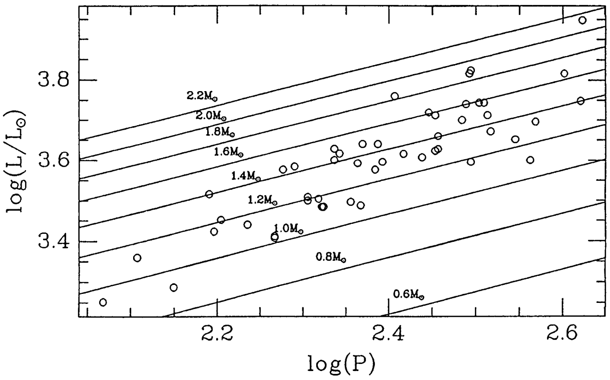

Figure 1. *The Log L shown plotted against the Log P from relation LPM, for Constant masses $M = 0.6\ M_\odot$ (0.2) $2.2\ M_\odot$, data from 49 stars (FGWC) have been included for comparison, the period is in days.*

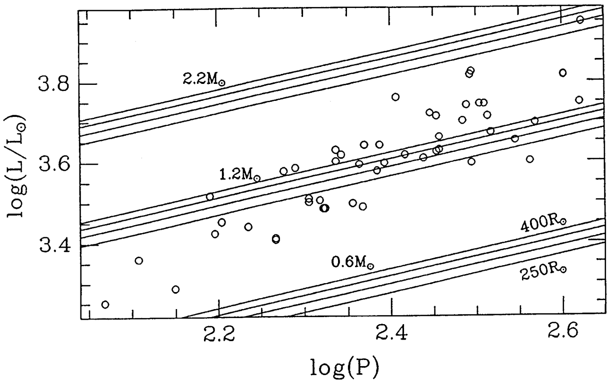

Figure 2. *LPMR, relation for $M = 0.6\ M_\odot$, $1.2\ M_\odot$ and $2.2\ M_\odot$ and $R = 250\ R_\odot$ (50) $400\ R_\odot$, data from 49 stars (FGWC) have been included for comparison, the period is in days.*

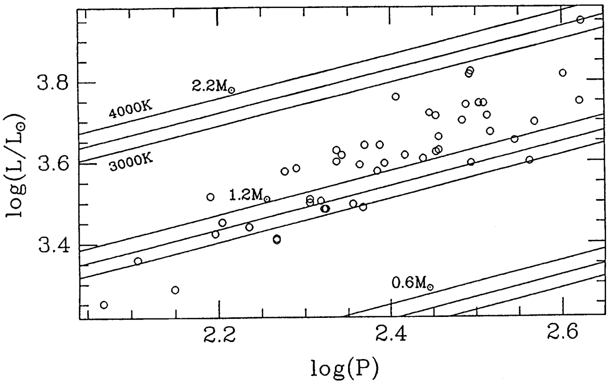

Figure 3. *From Equation (3), three set of, 3 lines for fixed masses $M = 0.6\ M_\odot$, $1.2\ M_\odot$ and $2.2\ M_\odot$ and $T = 3000$ K (500) 4000 K are plotted in Log L-Log P plane, data from 49 stars (FGWC) have been included for comparison, the period is in days.*

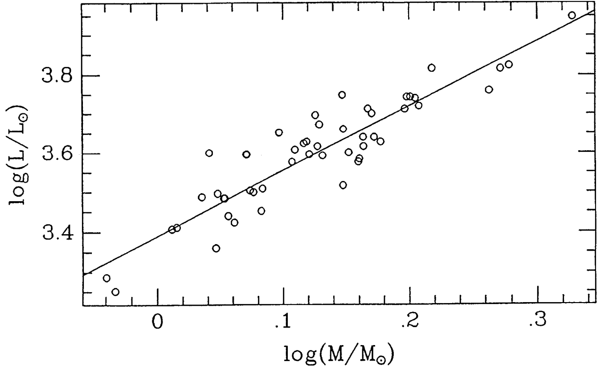

Figure 4. *Luminosity-Mass relation for data from 49 stars (FGWC). Mass is calculated from LPM, relation for a given luminosity and period of data, the line is a least square fit of result.*

4. Conclusion

In this paper it was shown that the Miras in the LMC, conform to the predictions of pulsation theory by the luminosity- period -Mass (LPM) and Luminosity-Period -Mass-Radius (LPMR) relations. As have been found by other investigators (FGWC) that Miras in particular the O-Miras, conform to the predictions of pulsation theory of Period-Luminosity-Color (PLC) relations, and Ostlie and Cox [8] have also derived the Period-Mass-Radius (PMR) relations from Mira variable stars. So, these results, together with the present work, suggest that the Miras are the physically well-defined class of objects which should be amenable to theoretical treatment in models of AGB evolution.

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References

1. *Bessell M.S., Wood P.R., and Lloyd Evans T. Monthly Notice Roy. Astron. Soc., **202**: 59 (1983).*
2. *Bohm-Vitense E. Zs. Ap., **46**: 108 (1958).*
3. *Feast M.W., Glass I.S., Whitelock P.A., and Catchople R.M. Monthly Notice Roy. Astron. Soc., **241**: 375 (1989).*
4. *Fox M.W. and Wood P.R. Astrophys. J., **259**: 198 (1982).*
5. *Glass I.S. and Feast M.W. Monthly Notice Roy. Astron. Soc., **199**: 245 (1982).*
6. *Iben I.Jr. and Truran J.W. Astrophys. J., **220**: 980 (1978).*
7. *Lioyd Evans T., Glass I.S., and Catchople R.M. Monthly Notices Roy. Astron. Soc., **231**: 773 (1988).*
8. *Ostlie D.A. and Cox A.N. Astrophys. J., **311**: 864 (1986).*
9. *Perl M. and Tuchman Y. Ibid., **360**: 554 (1990).*
10. *Stellingwerf R.F. Ibid., **195**: 441 (1975a).*
11. *Stellingwerf R.F. Ibid., **199**: 705 (1975b).*
12. *Takeuti M. Sci. Reports, Tohoku Unv., Ser. I, 62, 7 (1979).*
13. *Whitelock P.A. Monthly Notice Roy. Astron. Soc., **219**: 525 (1986).*