Journal of Sciences, Islamic Republic of Iran 31(4): 349 - 356 (2020) http://jsciences.ut.ac.ir
University of Tehran, ISSN 1016-1104

Constrained Supply Chain Scheduling Model Using Sequential Quadratic Programming Algorithm

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Received: 7 April 2019 / Revised: 26 April 2020 / Accepted: 24 June 2020

Abstract

Challenges such as advances in technology, demands of the global market, and limited warehouse spaces resort manufacturing industries to subcontracting. Subcontracting has been a considerable alternative in the manufacturing industries and is utilized as a strategic tool to diminish operation costs primarily to address the problem of scarcity when the firm faces a large demand on the commodity it supplies. The present study employed a mathematical model among firms engaging in subcontracting in search of an optimal schedule in the manufacture of the product and distribution of production time involved with an objective of obtaining a maximum profit. The constraints in the mathematical formulation included the total demand, processing capacity, available supply, processing rate, and time. The plausibility and the possible utility of the mathematical model has been explored employing sequential quadratic programming algorithm in the search of the optimal solutions.

Keywords: Subcontracting; Mathematical model; Constrained optimization; Nonlinear programming.

Introduction

Subcontracting, more commonly referred to as a sweating system, began between the late 1800s and early 1900s and the conventional subcontracted laborer was the tailor working in an apartment in New York City throughout the end of the nineteenth century [1]. The concept of sweating includes hiring out of the competing manufacturers to competing contractors the components of clothes, which in succession, given to laborers to be manufactured [2]. Over the years, manufacturing industries have been challenged by several technological advances and demands of the global market [3, 4] along with the shortage and limited warehouse spaces [5, 6]. Hence, concepts were designed and theories were formulated to address these problems [3], to create sustainable frameworks and to propose better policies [7, 8]. Moreover, models were built to optimize operation processes [6, 8-14] while employing several computational approaches and numerical algorithms [5, 6, 15, 16].

Nowadays, subcontracting has already been a

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considerable option in the manufacturing industry, a practice of assigning part of the obligations and tasks under a contract to another party. It is mainly caused by technological constraints where the manufacturer does not have the ability or expertise to produce the good, or capacity related limitations where the manufacturer has insufficient production capacity. It can be employed as a strategic tool to reduce operation cost and as means to hedge against the capacity shortage when facing a large demand [17], and subcontractors are employed by administrators who are assumed to have general liability for the completion and implementation within the given limitations and deadlines.

Several factors such as in-house production cost, subcontracting cost, delivery lead times, and demand priorities should be identified when subcontracting. With all these factors considered, along with an appropriate mathematically formulated model, the objective of maximizing the expected returns of the firm can be realized. An analytical scheduling model for a firm with an option of subcontracting specific demands of clients while taking into consideration the production cost is proposed on the assumptions that there are multiple identical parallel in-house machines with multiple subcontractors performing various production processes. Further, each job can either be processed in-house or can be outsourced to a subcontractor. As such, when the firm is given a set of orders, it should determine which job should be processed in-house or outsourced such that it will yield a maximum profit. Moreover, the mathematical model should determine the optimal scheduling of time per production stage and the distribution of finished products to the clients.

A scheduling model on subcontracting can be employed by manufacturing companies with objectives of maximizing revenue and minimizing cost while maintaining a good business operation relationship between clients and subcontractors. Consequently, the utility of the results of the mathematical formulation in this study allows developing companies to expand their production while considering their operation limits and meeting the demands of their respective clients. The proposed scheduling model will address the problem of manufacturing firms operating in a business environment under the assumptions specified in this study.

Materials and Methods

Mathematical Model Formulation

In this study, the following are the assumptions of the mathematical model: a) the firm has multiple parallel machines per stage, producing identical product in a given production stage. b) there is a finite number of clients requiring varying amounts of demand. c) at each production stage, the number of in-house and subcontractor machines varies. d) there are multiple subcontractors within each stage producing exactly the same product as the firm. e) the in-house and outsource product costs and processing time vary. f) no other external sources of the product at any stage of the production other than the subcontractors, and g) the supply is constant throughout all the stages. To better understand and visualize the presented problem under the given assumptions, a schematic diagram is shown in Figure 1.

The quantity x_{jk_0} is to be processed on the j^{th} stage of the production using the k_0^{th} in-house machine, $k_0 = 1,2,\ldots,m,$ while the quantity $\bar{x}_{jk_1}$ is to be processed on the j^{th} stage of the production using the k_1^{th} subcontractor machine, $k_1 = 1,2,\ldots,q$. The aggregate amount at each j^{th} processing stage to be supplied to the clients is s_j and is equal to the total available supply S. The allotted time for the completion of the j^{th} process or stage is denoted as t_j, and the allotted supply of finished product for the r^{th} client is denoted as y_r.

An analytical scheduling model with an objective of maximizing profit and minimizing cost given an option of subcontracting revealed a significant reduction to the objective value when the subcontracting option was used [18]. In subcontracting, the firm should maintain and satisfy due dates, focus on the time constraint and technological feasibility, and take into consideration the minimum use of possible resources [19]. As such, a firm can decide whether to produce in-house or outsource, assess the robustness of the policy, and identify which option utilizes most of the resources [20].

In the formulation of any mathematical program, the decision variables must be properly defined, the objective is functionally constructed, and the constraints are appropriately expressed. In this study, there are three types of decision variables and these include the quantity (x_{jk_0}) to be produced at the j^{th} stage using the m in-house machines or the quantity ($\bar{x}_{jk_1}$) using the q subcontractor machines, the processing time (t_j) at the j^{th} stage, and the allotted supply of finished product (y_r) to the r^{th} client. Since the objective is to maximize the profit level, it is necessary to define its two components, namely the revenue (z_1) and the cost (z_2). If c_r is the price that client r is willing to pay for every finished product, and the allocated supply for client r is y_r, for $r = 1,\ldots,f$, and if z_1 is the revenue function, then this revenue function can be expressed as

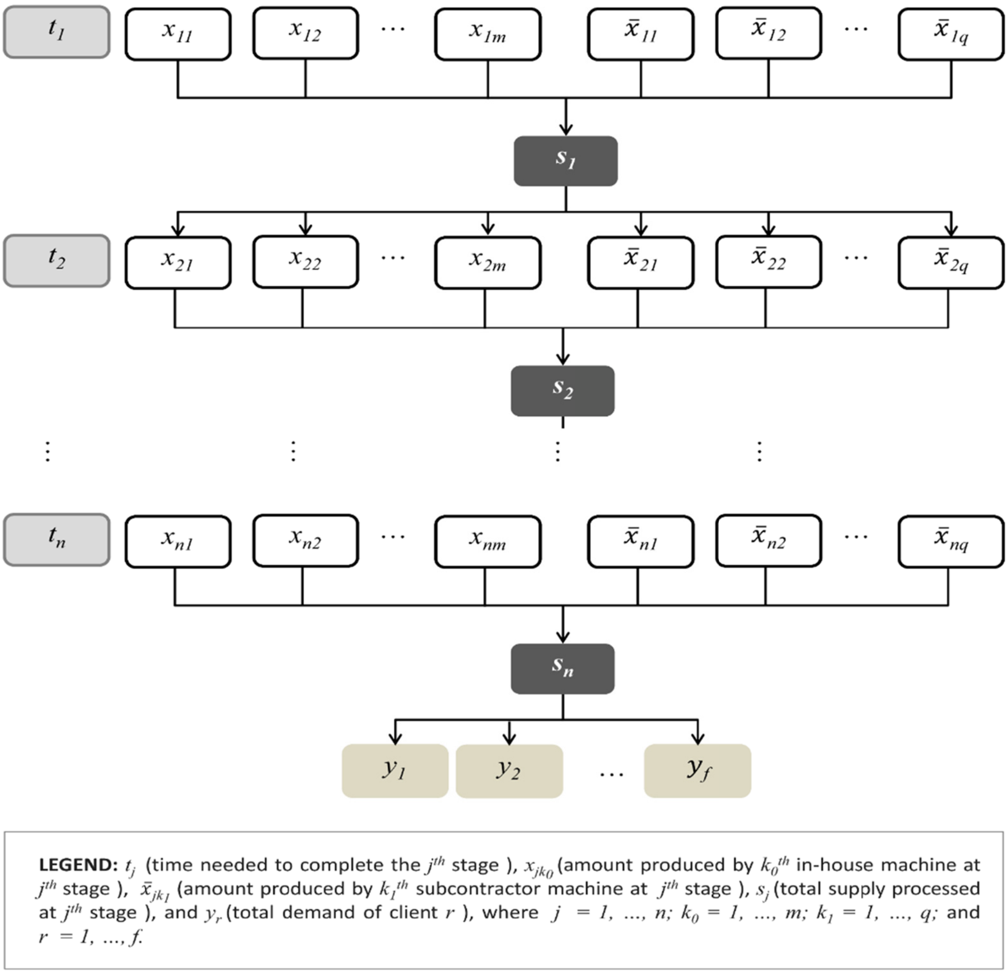

Figure 1. Schematic diagram of the production processes at different stages and the distribution of the manufactured units to the different clients using the in-house and subcontractor machines.

$$z_1 = \sum_{r=1}^{f} c_r y_r. \tag{1}$$

Considering that the production cost g_{jk_0} on the j^{th} stage of the production using the k_0^{th} in-house machine, and the production cost $\bar{g}_{jk_1}$ on the j^{th} stage of the production using the k_1^{th} subcontractor machine vary depending on which stage of the process and machine a product is in, the total production cost function z_2 can be represented as

$$z_2 = \sum_{j=1}^{n} \sum_{k_0=1}^{m} g_{jk_0} x_{jk_0} + \sum_{j=1}^{n} \sum_{k_1=1}^{q} \bar{g}_{jk_1} \bar{x}_{jk_1}. \tag{2}$$

Hence, the profit function $(z_1 - z_2)$, defined as revenue function minus the cost function, can be expressed as

$$z_1 - z_2 = \sum_{r=1}^{f} c_r y_r - \left(\sum_{j=1}^{n} \sum_{k_0=1}^{m} g_{jk_0} x_{jk_0} + \sum_{j=1}^{n} \sum_{k_1=1}^{q} \bar{g}_{jk_1} \bar{x}_{jk_1}\right). \tag{3}$$

The allotted quantity of finished product to be supplied to client r is y_r, the aggregate amount to be supplied to the clients is S, the amount to be produced at the j^{th} process is s_j, the total amount to be produced by the k_0^{th} in-house machine in the j^{th} process is x_{jk_0}, while the total amount to be produced by the k_1^{th} subcontractor machine in the j^{th} process is $\bar{x}_{jk_1}$, and the quantity demanded of client r is d_r. The allotted time for the production in the j^{th} process using both in-house and subcontractor machines is t_j, while T is the total allotted time for all the processes, p_{jk_0} is the processing time for the production in the j^{th} stage by the k_0^{th} in-house machine, and $\bar{p}_{jk_1}$ is the processing time for the production in the j^{th} stage by the k_1^{th} subcontractor machine.

The total demand constraint considers the combined allotted quantity of finished products among clients, and this should be equal to the total available supply since there will be a limited supply of the finished products. Thus, the total demand constraint is expressed as

$$\sum_{r=1}^{f} y_r = S. \tag{4}$$

The processing capacity is the combined amount of products processed through the in-house and subcontractor machines per stage j. This will be equivalent to S, which is constant throughout all the stages. These products will be aggregated and then distributed to the succeeding in-house and subcontractor machines. The processing capacity constraint will take the form of

$$\sum_{k_0=1}^{m} x_{jk_0} + \sum_{k_1=1}^{q} \bar{x}_{jk_1} = s_j = S, \text{ for } j = 1, \dots, n. \tag{5}$$

Since there will be limited supply for the finished products, specific demands of the clients will be prioritized depending on their rate of return for these products, cases are either all or some of the demands of a specific client will be met. This allotted supply constraint can be expressed as

$$y_r \leq d_r, \text{ for } r = 1, \dots, f. \tag{6}$$

Moreover, the mathematical formulation will consider processing time for both in-house and subcontractor machines. The rate constraint is defined as the sum of the total amount produced by each of the machines divided by their respective processing time multiplied to the allotted time in the completion of a process in a particular stage. This rate constraint is expressed as

$$t_j \left(\sum_{k_0=1}^{m} \frac{x_{jk_0}}{p_{jk_0}} + \sum_{k_1=1}^{q} \frac{\bar{x}_{jk_1}}{\bar{p}_{jk_1}} \right) \leq s_j = S, \text{ for } j = 1, \dots, n. \tag{7}$$

The sum of the allotted time in all the stages of production is less than or equal to the total allotted time for the entire production process, and since the problem seeks to determine the allotted time (t_j) for each stage j, the aggregate of the allotted time among the n stages will be less than or equal to the total allotted time (T), and this time constraint is represented by

$$\sum_{j=1}^{n} t_j \leq T. \tag{8}$$

The processing time and allotted time at stage j are greater than zero since any in-house or subcontractor machine requires non-zero unit of time in the completion of the job at any stage. These variables are not necessarily integers, and these restrictions are expressed as

$$p_{jk_0},\ \bar{p}_{jk_1}, t_j > 0. \tag{9}$$

In this paper, all the remaining decision variables assume non-negative real values since these are the quantities allotted and to be distributed to the respective clients. Further, the allocated supply for client r can possibly take a value of zero. These non-negativity restrictions can be expressed as

$$x_{jk_0},\ \bar{x}_{jk_1}, y_r \geq 0. \tag{10}$$

Hence, the final mathematical model formulation for this problem is
Minimize

$$z_2 - z_1 = \sum_{j=1}^{n} \sum_{k_0=1}^{m} g_{jk_0} x_{jk_0} + \sum_{j=1}^{n} \sum_{k_1=1}^{q} \bar{g}_{jk_1} \bar{x}_{jk_1} - \sum_{r=1}^{f} c_r y_r \tag{11}$$

subject to constraints defined as Equations 4-10.

Given this mathematical model formulation, several optimization methods can be utilized in the search of the optimal solution. In this paper, sequential quadratic programming was employed.

Sequential Quadratic Programming Algorithm

Sequential quadratic programming (SQP) methods belong to the most powerful nonlinear programming algorithms used in solving differentiable nonlinear programming problems [21]. The basic idea behind the SQP algorithm is to formulate and solve a quadratic programming sub-problem in each iteration, which is obtained by linearizing the constraints and quadratically approximating the Lagrangian function [22]. The advantage of using SQP, compared to the traditional Newton method, is that it allows for a systematic and natural way of selecting the active set of constraints and in addition, through the use of the merit function, the convergence process may be controlled [23].

In order to determine the optimal solution of the proposed mathematical formulation in this paper, we generated MATLAB codes to obtain the solution. In particular, we made use of the FMINCON function found in the optimization toolbox of MATLAB, to represent the specified mathematical model shown in Equation 11.

Initial Value Specification

An estimated initial starting value for the decision variables is required in an iterative procedure. In fact, starting value specification is one of the most difficult problems encountered in estimating parameters of nonlinear models [21], and the problem of indicating initial values of the decision variables can be solved with the proper understanding of the definition of the parameters in the context of the phenomenon being modeled. Wrong starting values result in longer iteration, greater execution time, non-convergence of the iteration, and possible convergence to an unwanted local minimum sum of squares residuals [24]. One of the frequently employed methods in specifying initial values in search of an optimal solution is to find these initial values in the basin of attraction of a global minimum.

A basin of attraction is the set of points in the space of system variables such that initial conditions chosen in this set dynamically evolve to a particular attractor [25]. It refers to the collection of initial conditions leading to long-time behavior that approaches the attractor(s) of a dynamical system [26]. Also, an attractor is a set of states, invariant under the dynamics, towards which neighboring states in a given basin of attraction asymptotically approach in the course of dynamic evolution. It is defined as the smallest unit which cannot be itself decomposed into two or more attractors with distinct basins of attraction [25].

In this paper, since the optimal solution is expected to be in the basin of attraction, the global minimum is one of the possible initial values set in an attempt to approach the optimal solution [26], and was the option considered in specifying the initial values.

Results and Discussion

As an application of the proposed mathematical formulation in this paper, several cases were constructed. In the formulation of the corresponding mathematical program for a particular case, the user is required to provide specific information including the number (m) of in-house machines and number (q) of subcontractor machines involved per stage or process, number (n) of stages or processes, the total number (f) of clients, total amount (S) of products to be produced in a given total allotted time (T).

The augmented matrix representation (X) of the decision variables x_{jk_0} and $\bar{x}_{jk_1}$ is

$$X = \left[x_{jk_0} \middle| \bar{x}_{jk_1}\right] = \left[\begin{array}{cccc|cccc} x_{11} & x_{12} & \cdots & x_{1m} & \bar{x}_{11} & \bar{x}_{12} & \cdots & \bar{x}_{1q} \\ x_{21} & x_{22} & \cdots & x_{2m} & \bar{x}_{21} & \bar{x}_{22} & \cdots & \bar{x}_{2q} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} & \bar{x}_{n1} & \bar{x}_{n2} & \cdots & \bar{x}_{nq} \end{array}\right].$$

The vector notation of the decision variable t_j is $[t_1, t_2, \dots, t_n]$, while the decision variable y_r is $\left[y_1, y_2, \dots, y_f\right]$. Hence, the cardinality of all the decision variables involved in the formulation is $n \text{ x } (m + q + 1) + f$.

The user is also required to provide values of the following matrices: P, G, D, and C. The processing time (p_{jk_0}) of in-house machines augmented with the processing time ($\bar{p}_{jk_1}$) of subcontractor machines will be denoted as matrix P. The production cost (g_{jk_0}) of in-house machines augmented with the production cost ($\bar{g}_{jk_1}$) of subcontractor machines will define matrix G. The quantities demanded by and the returns of the clients are denoted as matrices D and C, respectively. These matrices P, G, D, and C are illustrated as follows:

$$P = \left[p_{jk_0} \middle| \bar{p}_{jk_1}\right] = \left[\begin{array}{cccc|cccc} p_{11} & p_{12} & \cdots & p_{1m} & \bar{p}_{11} & \bar{p}_{12} & \cdots & \bar{p}_{1q} \\ p_{21} & p_{22} & \cdots & p_{2m} & \bar{p}_{21} & \bar{p}_{22} & \cdots & \bar{p}_{2q} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nm} & \bar{p}_{n1} & \bar{p}_{n2} & \cdots & \bar{p}_{nq} \end{array}\right]$$

$$G = \left[g_{jk_0} \middle| \bar{g}_{jk_1}\right] = \left[\begin{array}{cccc|cccc} g_{11} & g_{12} & \cdots & g_{1m} & \bar{g}_{11} & \bar{g}_{12} & \cdots & \bar{g}_{1q} \\ g_{21} & g_{22} & \cdots & g_{2m} & \bar{g}_{21} & \bar{g}_{22} & \cdots & \bar{g}_{2q} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nm} & \bar{g}_{n1} & \bar{g}_{n2} & \cdots & \bar{g}_{nq} \end{array}\right]$$

$$D = [d_1, \quad d_{2,} \quad \cdots, \quad d_f]$$

$$C = [c_1, \quad c_{2,} \quad \cdots, \quad c_f]$$

Illustration A

A total amount of 15000 is needed to be produced in 500 days. A manufacturing firm has two in-house and three subcontractor machines. Each product has to undergo four stages before it is distributed to the six clients. The processing time (P), production cost (G), demand (D), and return (C) matrices are given as

$$P = \left[\begin{array}{cc|ccc} 75 & 75 & 80 & 75 & 90 \\ 80 & 75 & 60 & 65 & 70 \\ 85 & 90 & 100 & 120 & 110 \\ 70 & 80 & 85 & 85 & 70 \end{array}\right]$$

$$G = \begin{bmatrix} 160 & 150 & | & 400 & 450 & 350 \\ 200 & 170 & | & 600 & 400 & 300 \\ 180 & 170 & | & 400 & 350 & 250 \\ 200 & 130 & | & 320 & 320 & 225 \end{bmatrix}$$

$$D = [3000 \quad 2800 \quad 3500 \quad 3000 \quad 3300 \quad 2000]$$

$$C = [1250 \quad 1100 \quad 1300 \quad 1000 \quad 1050 \quad 1000]$$

The schematic representation of this problem is illustrated in Figure 2A. Given S = 15000 and T = 500, the mathematical programming formulation of this problem is

Minimize

$$\begin{aligned}(z_2 - z_1) = {} & -1250y_1 - 1100y_2 - 1300y_3 + 1000y_4 \\ & - 1050y_5 - 1000y_6 + 160x_{11} \\ & + 150x_{12} + 400\bar{x}_{11} + 450\bar{x}_{12} \\ & + 350\bar{x}_{13} + 200x_{21} + 170x_{22} \\ & + 600\bar{x}_{21} + 400\bar{x}_{22} + 300\bar{x}_{23} \\ & + 180x_{31} + 170x_{32} + 400\bar{x}_{31} \\ & + 350\bar{x}_{32} + 250\bar{x}_{33} + 200\,x_{41} \\ & + 130x_{42} + 320\bar{x}_{41} + 320\bar{x}_{42} \\ & + 225\bar{x}_{43}\end{aligned}$$

subject to

$$x_{11}+x_{12} + \bar{x}_{11} + \bar{x}_{12} + \bar{x}_{13} = 15000$$
$$x_{21} + x_{22} + \bar{x}_{21} + \bar{x}_{22} + \bar{x}_{23} = 15000$$
$$x_{31} + x_{32} + \bar{x}_{31} + \bar{x}_{32} + \bar{x}_{33} = 15000$$
$$x_{41} + x_{42} + \bar{x}_{41} + \bar{x}_{42} + \bar{x}_{43} = 15000$$
$$t_1 + t_2 + t_3 + t_4 \leq 500$$
$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 = 15000$$
$$y_1 \leq 3000$$
$$y_2 \leq 2800$$
$$y_3 \leq 3500$$
$$y_4 \leq 3000$$
$$y_5 \leq 3300$$
$$y_6 \leq 2000$$
$$t_1\left(\frac{x_{11}}{75} + \frac{x_{12}}{75} + \frac{\bar{x}_{11}}{80} + \frac{\bar{x}_{12}}{75} + \frac{\bar{x}_{13}}{90}\right) = 15000$$
$$t_2\left(\frac{x_{21}}{80} + \frac{x_{22}}{75} + \frac{\bar{x}_{21}}{60} + \frac{\bar{x}_{22}}{65} + \frac{\bar{x}_{23}}{70}\right) = 15000$$
$$t_3\left(\frac{x_{31}}{85} + \frac{x_{32}}{90} + \frac{\bar{x}_{31}}{100} + \frac{\bar{x}_{32}}{120} + \frac{\bar{x}_{33}}{110}\right) = 15000$$
$$t_4\left(\frac{x_{41}}{70} + \frac{x_{42}}{80} + \frac{\bar{x}_{41}}{85} + \frac{\bar{x}_{42}}{85} + \frac{\bar{x}_{43}}{70}\right) = 15000$$
$$x_{jk_0},\ \bar{x}_{jk_1}, y_r \geq 0$$
$$t_j > 0$$
$$k_0 = 1,2;\ k_1 = 1,2,3;\ j = 1,2,3,4;\ r = 1,2,3,4,5,6$$

A maximum profit of 7945000 was obtained in this case, and the optimal solution showed that the total amount of 15000 should be processed by the second in-house machines in the four stages. Further, a total of 75 days should be allotted in each of the first and second stages of the production, while a total of 90 and 80 days for the third and fourth stages, respectively. The optimal solution revealed the amount to be supplied for the first, second, third, fourth, fifth, and sixth clients are 3000, 2800, 3500, 1437, 3300, and 963 units, respectively.

Illustration B

A certain product has to undergo three stages in its production processes. A firm has a total of three in-house machines and one subcontractor machine and can produce a total amount of 100 to be distributed to its three clients five days from now. The matrices for processing time (*P*), production cost (*G*), demand (*D*), and return (*C*) are given as

$$P = \begin{bmatrix} 1.5 & 1.5 & 1 & | & 1 \\ 2.5 & 2.5 & 1.5 & | & 2 \\ 1.5 & 1.5 & 0.5 & | & 0.5 \end{bmatrix}$$

$$G = \begin{bmatrix} 60 & 55 & 62 & | & 50 \\ 55 & 70 & 65 & | & 60 \\ 60 & 50 & 55 & | & 40 \end{bmatrix}$$

$$D = [45 \quad 45 \quad 40]$$

$$C = [260 \quad 210 \quad 230]$$

The diagram describing the representation of this problem is shown in Figure 2B. Given a capacity of an amount of 100 and five days completion time, the problem can be mathematically formulated as

Minimize

$$\begin{aligned}(z_2 - z_1) = {} & -260y_1 - 210y_2 - 230y_3 + 60x_{11} \\ & + 55x_{12} + 62x_{13} + 50\bar{x}_{11} + 55x_{21} \\ & + 70x_{22} + 65x_{23} + 60\bar{x}_{21} + 60x_{31} \\ & + 50x_{32} + 55x_{33} + 40\bar{x}_{31}\end{aligned}$$

subject to

$$x_{11} + x_{12} + x_{13} + \bar{x}_{11} = 100$$
$$x_{21} + x_{22} + x_{23} + \bar{x}_{21} = 100$$
$$x_{31} + x_{32} + x_{33} + \bar{x}_{31} = 100$$
$$t_1 + t_2 + t_3 \leq 5$$
$$y_1 + y_2 + y_3 = 100$$
$$y_1 \leq 45$$
$$y_2 \leq 45$$
$$y_3 \leq 40$$
$$t_1\left(\frac{x_{11}}{1.5} + \frac{x_{12}}{1.5} + x_{13} + \bar{x}_{11}\right) = 100$$

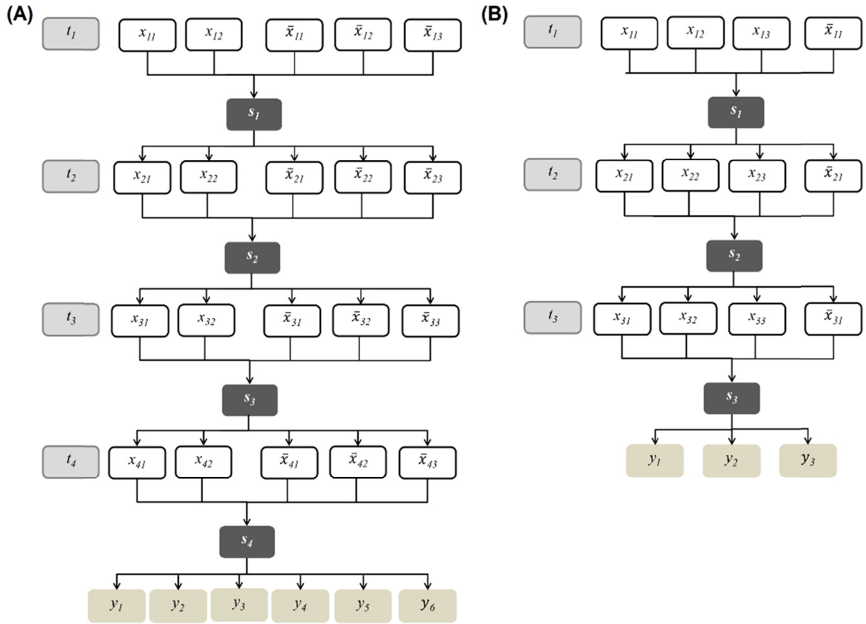

Figure 2. Schematic representations of illustrated cases using constrained supply chain scheduling.

$$t_2\left(\frac{x_{21}}{2.5}+\frac{x_{22}}{2.5}+\frac{x_{23}}{1.5}+\frac{\bar{x}_{21}}{2}\right)=100$$
$$t_3\left(\frac{x_{31}}{1.5}+\frac{x_{32}}{1.5}+\frac{x_{33}}{0.5}+\frac{\bar{x}_{31}}{0.5}\right)=100$$
$$x_{jk_0},\ \bar{x}_{jk_1}, y_r \geq 0$$
$$t_j > 0$$
$$k_0 = 1,2,3;\ k_1 = 1;\ j = 1,2,3;\ r = 1,2,3$$

An optimal value of 9550 defines the maximum profit for this problem. The optimal solution revealed that the total amount of 100 should be produced by the subcontractor in the first stage of production, then all of the 100 be processed using the first in-house machine in the second stage and will then be eventually assigned to the subcontractor on the third stage of production. A day is allotted for the completion of the first process, while a total of 2.5 days for the second process, and a half-day for the third process. The maximum profit will be obtained if the total supply of 100 will be distributed as 45, 15, and 40 to the first, second, and third clients, respectively.

Conclusion

With the shortage of equipment and limited warehouse spaces, along with technological advances and high demands of the global market, most manufacturing industries rely on subcontracting. The present study employed a mathematical program which can be utilized by firms engaging in subcontracting in search for an optimal schedule in the distribution of the amount of production, the time needed at different stages, and amount to supply in order to realize an objective of maximizing the profit. Although there are existing computational approaches in handling supply chain related processes, the present study employed a mathematical program which assumed that the firm has its own machines and considered subcontracting as an external source of the products to be processed and eventually distributed in order to meet the demands of its clients. The constraints in the formulated model included the total demand, processing capacity, allotted supply, rate, and time. The FMINCON function in MATLAB along with SQP algorithm was utilized to assess the feasibility of the problem and eventually to determine the optimal schedule of some identified cases.

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