Journal of Sciences, Islamic Republic of Iran 32(4): 349 - 357 (2021) http://jsciences.ut.ac.ir
University of Tehran, ISSN 1016-1104

Introduction to Reliability for Conditional Stress-Strength Parameter

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Received: 10 Nov 2020 / Revised: 8 Nov 2021 / Accepted: 21 Dec 2021

Abstract

In this article, a new proper and favorite stress-strength parameter has been introduced. The maximum likelihood and uniformly minimum variance unbiased estimators of the purposed parameter have been derived for the Exponential distribution. Moreover, the nonparametric estimator of this parameter has also been obtained as well as some important properties of this estimator. A simulation study and the analysis of a real data set have been done for illustrative purposes.

Keywords: Stress-Strength parameter; Multivariate Delta method; UMVUE; Consistent estimator.

Introduction

The well-known stress-strength parameter denoted by R, is defined as

$$R = P(X > Y). \quad (1.1)$$

After the first definition, its application had a rapid growth in different areas such as reliability, biostatistics, quality control and engineering. In a clinical study, Kotz et al. [1], have considered Y and X as the outcomes of a treatment and a control group respectively, in this case the quantity $(1 - R)$ measures the treatment efficiency. Some applications of R have been studied in Ventura and Racugno [2]. In reliability context, Y is the strength of a component which is subject to the stress X. Therefore, R is the chance that the system fails and $(1 - R)$ measures the probability of system performance. See Rezaei et al. [3] for a list of customary distributions in the study of R.

By the time lapse, more modern and advancer methods for estimating the stress-strength parameter have been brought to the literature. For instance, Hassan et al. [4] have estimated this parameter based on upper record values where Almarashi, et al. [5] have applied some different procedures for the systems with Weibull distributed components.

In many situations, the inspector has the valuable information that X and Y are greater than some real-valued levels, a and b, particularly, for X and Y as the lifetimes of two components of a system. Suppose that by using the above mentioned information, it is desired to have some inferences about $R = P(X > Y)$. For clarification, consider a car with two active components, an engine and a brakes whose lifetimes are denoted by Y and X, respectively. It is obvious that before the car starts, it works correctly and with probability (1.1), iff $\{X > Y\}$. If the car has been driven for one hour, a reasonable measure of reliability of the car performance, is $P(X > Y | X > 1, Y > 1)$. Furthermore, consider the situation that the engine has been switched on 15 minutes before driving, then the appropriate measure of reliability is $P(X > Y | X > 1, Y > 1.25)$. Thus, we have been motivated to introduce the conditional stress-strength parameter denoted by $R^{|a,b}$.

There are some distributions which are more customary in calculating and estimation of the parameter R, these distributions will have been used for

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studying $R^{|a,b}$, as well. The most well-known distribution in lifetime studies is Exponential, which in studying $R^{|a,b}$ has been considered at first. For different extensions of the reliability measure see Mirjalili et al. [6], Kazemi [7], Khalifeh et al. [8]. We also compute $R^{|a,b}$ for Gamma distribution very briefly. For some studies on parameter R for Gamma distributed components see Krishnamoorthy et al. [9], Huang et al. [10] and Chen and Ye [11].

This paper is organized as follows. A general formula for computing the conditional stress-strength parameter is presented in Section 2. This section also includes the study and estimation of $R^{|a,b}$ for several special and usual situations e.g., in the case of *iid* components, Exponential and Gamma distributions, as well as some nonparametric inferences. Certain simulation study and parameter estimation for a real data set have been carried out in Section 3. Finally, Section 4 is devoted to some practical extensions of R and $R^{|a,b}$.

The numerical calculations and corresponding programs has been written by the R software version 4.1.0.

Conditional Stress-strength Parameter

The conditional stress-strength parameter is defined as

$$R^{|a,b} = P(X > Y|X > a, Y > b). \qquad (2.1)$$

Note that the usual R is an special case of this quantity for $a = b = -\infty$.

As the first task, a general formula for computing (1.2) will have been presented.

Proposition 1: suppose that X and Y are two independent continuous random variables. The conditional stress-strength parameter (2.1), may be calculated as:

$$R^{|a,b} = \begin{cases} \frac{\bar{F}_Y(b)-\int_b^{+\infty} F_X(y)f_Y(y)\,dy}{\bar{F}_X(b)\ \bar{F}_Y(b)} & a = b \\ \frac{\bar{F}_Y(b)-\int_b^{+\infty} F_X(y)f_Y(y)\,dy}{\bar{F}_X(a)\ \bar{F}_Y(b)} & a < b, \\ \frac{\int_a^{+\infty} F_Y(y)f_X(y)\,dy-F_Y(b)\bar{F}_X(a)}{\bar{F}_X(a)\ \bar{F}_Y(b)} & a > b \end{cases} \qquad (2.2)$$

where $\bar{F}(x) = 1 - F(x)$.

Proof. By definition, $R^{|a,b} = \frac{P(X>Y,X>a,Y>b)}{P(X>a,Y>b)}$, so by independence of X and Y, the dominator may simplified as $P(X > a, Y > b) = \bar{F}_X(a)\ \bar{F}_Y(b)$. For computing the nominator we have

$P(X > Y, X > a, Y > b) = P\big((X, Y) \in \boldsymbol{A}\big) = \iint_{\boldsymbol{A}} f_X(x)f_Y(y)\,dx\,dy$,

where $\boldsymbol{A} = \{(x, y)|x > y, x > a, y > b\}$. The set $\boldsymbol{A}$ will be divided to $\boldsymbol{A}_{a\le b}$ and $\boldsymbol{A}_{a>b}$ for the cases $a \le b$ and $a > b$, as follows:

$\boldsymbol{A}_{a\le b} = \{(x, y)|x > y, x > b, y > b\}$, $\boldsymbol{A}_{a>b} = \{(x, y)|b < y < x, x > a\}$.

Therefore,

$\iint_{\boldsymbol{A}_{a\le b}} f_X(x)f_Y(y)\,dx\,dy = \int_b^{+\infty} f_Y(y)(1 - F_X(y))\,dy = \bar{F}_Y(b) - \int_b^{+\infty} F_X(y)f_Y(y)\,dy$,

and

$$\iint\limits_{\boldsymbol{A}_{a>b}} f_X(x)f_Y(y)\,dx\,dy = \int_a^{+\infty} f_X(x)(F_Y(x) - F_Y(b))\,dx = \int_a^{+\infty} F_Y(x)f_X(x)\,dx - F_Y(b)\bar{F}_X(a)$$

which completes the proof. ■

Remark 1. By definition (2.1), one may expect that $R^{|a,b}$ be an increasing (decreasing) function of the arguments $a(b)$. We will verify it for $a < b$, in terms of a:

$$\frac{\partial R^{|a,b}}{\partial a} = \frac{\left(\bar{F}_Y(b)-\int_b^{+\infty} F_X(y)f_Y(y)\,dy\right)f_X(a)}{\left(\bar{F}_X(a)\right)^2 \bar{F}_Y(b)} = \frac{\bar{F}_Y(b)-\int_b^{+\infty} F_X(y)f_Y(y)\,dy}{\bar{F}_X(a)\,\bar{F}_Y(b)}\frac{f_X(a)}{\bar{F}_X(a)} = R^{|a,b}\frac{f_X(a)}{\bar{F}_X(a)} \ge 0. \ ■$$

In the following corollary, $R^{|a,b}$ is computed for identically distributed X and Y.

Corollary 1. Suppose that the continuous random variables X and Y are independent and identically distributed with pdf $f(.)$ and cdf $F(.)$. Then,

$$R^{|a,b} = \begin{cases} \frac{1}{2} & a = b \\ \frac{\bar{F}(b)}{2\,\bar{F}(a)} & a < b. \\ \frac{1+F(a)-2F(b)}{2\bar{F}(b)} & a > b \end{cases} \qquad (2.3)$$

Proof. By substituting $\int_a^{+\infty} F(y)f(y)\,dy = \frac{1-F^2(a)}{2}$ in (2.2), we arrive at (2.3). ■

Remark 2. For independent and identically distributed X and Y, by using (2.3) it is immediate that:

$$R^{|a,b;a<b} < R^{|a,a} < R^{|a,b;a>b}. \qquad ■ \qquad (2.4)$$

From statistical point of view, (2.4) seems to be rational. The values of $R^{|a,b}$ has been computed for some well-known continuous distributions and various values of a and b. The results have been figured out in the Table 1, which confirm (2.4) as was expected.

Table 1. $R^{|a,b}$ for some identically distributed variables. For beta distribution the values of a and b have been divided to 10.

Distribution/(a,b)	(1,2)	(2,5)	(3,1)	(7,4)
$\boldsymbol{N(0,1)}$	0.072	6.3e-06	0.99	1
$\boldsymbol{Gamma(3,0.5)}$	0.47	0.3	0.59	0.76
$\boldsymbol{Beta(3,1)/\div 10}$	0.49	0.44	0.51	0.65
$\boldsymbol{F(2,4)}$	0.28	0.16	0.82	0.78

Estimation for Exponential Distribution

In this subsection, the measure (2.2) has been evaluated for Exponentially distributed components. First of all, the multivariate Delta method has been recalled by the following lemma.

Lemma 1. Suppose that $\{\boldsymbol{X}_n\}_{n=1}^{\infty}$ be a sequence of random vectors where $\boldsymbol{X}_n \to \boldsymbol{N}_p\,(\boldsymbol{\mu},\boldsymbol{\Sigma})$ in distribution. Also let $g(\boldsymbol{x}): R^p \to R$ be continuous in the first partial derivatives and $\tau^2 = \boldsymbol{\nabla}^T\,\boldsymbol{\Sigma}\,\boldsymbol{\nabla} > 0$, where $\boldsymbol{\nabla} = \frac{\partial g(\boldsymbol{\mu})}{\partial \boldsymbol{\mu}}$. Then, $\frac{g(X_n)-g(\mu)}{\tau} \to N\,(0,1)$. ■

Example 1: suppose that X and Y are two independent exponential random variables with means $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$, respectively. Then,

$$\int_b^{+\infty} F_X(y) f_Y(y)\, dy = \bar{F}_Y(b) - \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-b\,(\lambda_1+\lambda_2)}.$$

Thus, the corresponding stress-strength parameter expressed in equation (2.2) is

$$R^{|a,b} = \begin{cases} \frac{\lambda_2}{\lambda_1+\lambda_2} & a=b \\ \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-\lambda_1\,(b-a)} & a<b. \\ 1-\frac{\lambda_1}{\lambda_1+\lambda_2} e^{-\lambda_2\,(a-b)} & a>b \end{cases} \tag{2.5}$$

Let $X_1,\dots,X_n \sim E(\lambda_1)$ and $Y_1,\dots,Y_m \sim E(\lambda_2)$, the MLE's for parameters have been denoted by $\hat{\lambda}_1 = \frac{1}{\bar{X}}$, $\hat{\lambda}_2 = \frac{1}{\bar{Y}}$. Since MLE's are invariant, the MLE of (2.5) will become

$$\hat{R}^{|a,b} = \begin{cases} \frac{\hat{\lambda}_2}{\hat{\lambda}_1+\hat{\lambda}_2} & a=b \\ \frac{\hat{\lambda}_2}{\hat{\lambda}_1+\hat{\lambda}_2} e^{-\hat{\lambda}_1\,(b-a)} & a<b. \\ 1-\frac{\hat{\lambda}_1}{\hat{\lambda}_1+\hat{\lambda}_2} e^{-\hat{\lambda}_2\,(a-b)} & a>b \end{cases} \tag{2.6}$$

For $\hat{\boldsymbol{\lambda}} = (\hat{\lambda}_1,\hat{\lambda}_2)^T$, $\boldsymbol{\lambda} = (\lambda_1,\lambda_2)^T$ and $\boldsymbol{\Sigma} = \boldsymbol{I^{-1}}(\boldsymbol{\lambda})$ as the inverse of Fisher's information $\boldsymbol{I}(\boldsymbol{\lambda})$, we have $\hat{\boldsymbol{\lambda}} \to N_2(\boldsymbol{\lambda},\boldsymbol{\Sigma})$, as n and m tend to infinity and by the assumption that $\frac{n}{m} \to p$ for some $0 < p < +\infty$. In Lemma 1, calculate $\boldsymbol{\Sigma}$, let $\boldsymbol{X}_n = \hat{\boldsymbol{\lambda}}$, and $g(\cdot)$ as below:

$$\boldsymbol{I}(\boldsymbol{\lambda}) = \begin{bmatrix} \frac{n}{\lambda_1^2} & 0 \\ 0 & \frac{m}{\lambda_2^2} \end{bmatrix}, \boldsymbol{\Sigma} = \begin{bmatrix} \frac{\lambda_1^2}{n} & 0 \\ 0 & \frac{\lambda_2^2}{m} \end{bmatrix}, \quad g(x_1,x_2) = \begin{cases} \frac{x_2}{x_1+x_2} & a=b \\ \frac{x_2}{x_1+x_2} e^{-x_1\,(b-a)} & a<b. \\ 1-\frac{x_1}{x_1+x_2} e^{-x_2\,(a-b)} & a>b \end{cases}$$

The partial derivatives of $g(\cdot)$ will become:

$$\frac{\partial g(x_1,x_2)}{\partial x_1} = \begin{cases} \frac{-x_2}{(x_1+x_2)^2} & a=b \\ \frac{-x_2}{(x_1+x_2)^2} e^{-x_1\,(b-a)}(1+(b-a)(x_1+x_2)) & a<b \\ \frac{-x_2}{(x_1+x_2)^2} e^{-x_2\,(a-b)} & a>b \end{cases}$$

$$\frac{\partial g(x_1,x_2)}{\partial x_2} = \begin{cases} \frac{x_1}{(x_1+x_2)^2} & a=b \\ \frac{x_1}{(x_1+x_2)^2} e^{-x_1\,(b-a)} & a<b \\ \frac{x_1}{(x_1+x_2)^2} e^{-x_2\,(a-b)}(1+(a-b)(x_1+x_2)) & a>b \end{cases}$$

The computation of $\boldsymbol{\nabla}^T\,\boldsymbol{\Sigma}\,\boldsymbol{\nabla}$ for $\boldsymbol{\nabla} = \left(\frac{\partial g(\lambda_1,\lambda_2)}{\partial \lambda_1}, \frac{\partial g(\lambda_1,\lambda_2)}{\partial \lambda_2}\right)^T$ results in

$$\boldsymbol{\nabla}^T\,\boldsymbol{\Sigma}\,\boldsymbol{\nabla} = \left[\frac{\partial g(\lambda_1,\lambda_2)}{\partial \lambda_1}\right]^2 \frac{\lambda_1^2}{n} + \left[\frac{\partial g(\lambda_1,\lambda_2)}{\partial \lambda_2}\right]^2 \frac{\lambda_2^2}{m}. \tag{2.7}$$

For the cases that $a=b$, $a<b$ and $a>b$, denote (2.7) by σ_1^2, σ_2^2 and σ_3^2, respectively. After some calculations we have

$$\sigma_1^2 = \frac{n+m}{nm} \frac{\lambda_1^2\lambda_2^2}{(\lambda_1+\lambda_2)^4}, \tag{2.8}$$

$$\sigma_2^2 = \frac{\lambda_1^2\lambda_2^2}{(\lambda_1+\lambda_2)^4} e^{-2\lambda_1\,(b-a)} \left(\frac{(1+(b-a)(\lambda_1+\lambda_2))^2}{n} + \frac{1}{m}\right), \tag{2.9}$$

and

$$\sigma_3^2 = \frac{\lambda_1^2\lambda_2^2}{(\lambda_1+\lambda_2)^4} e^{-2\lambda_2\,(a-b)} \left(\frac{(1+(a-b)(\lambda_1+\lambda_2))^2}{m} + \frac{1}{n}\right). \tag{2.10}$$

Finally, we arrive at the following asymptotically normalized estimators of $R^{|a,b}$:

$$\begin{cases} \frac{\hat{R}^{|a,b}-R^{|a,b}}{\sigma_1} \to N(0,1) & a=b \\ \frac{\hat{R}^{|a,b}-R^{|a,b}}{\sigma_2} \to N(0,1) & a<b \\ \frac{\hat{R}^{|a,b}-R^{|a,b}}{\sigma_3} \to N(0,1) & a>b \end{cases} \quad (2.11)$$

By using the equations in (2.11), some $100(1-\alpha)\%$ asymptotic confidence interval of $R^{|a,b}$ may be constructed as below:

$$\begin{cases} R^{|a,b} \in (\hat{R}^{|a,b} - z_{1-\frac{\alpha}{2}}\hat{\sigma}_1 , \hat{R}^{|a,b} + z_{1-\frac{\alpha}{2}}\hat{\sigma}_1) & a=b \\ R^{|a,b} \in (\hat{R}^{|a,b} - z_{1-\frac{\alpha}{2}}\hat{\sigma}_2 , \hat{R}^{|a,b} + z_{1-\frac{\alpha}{2}}\hat{\sigma}_2) & a<b \\ R^{|a,b} \in (\hat{R}^{|a,b} - z_{1-\frac{\alpha}{2}}\hat{\sigma}_3 , \hat{R}^{|a,b} + z_{1-\frac{\alpha}{2}}\hat{\sigma}_3) & a>b \end{cases} \quad (2.12)$$

In Equation (2.12), $\hat{\sigma}_i^2, i=1,2,3$ are the analogues of $\sigma_i^2, i=1,2,3$ in Equations (2.8)-(2.10), which are obtained by substituting $\hat{\lambda}_1$ and $\hat{\lambda}_2$ instead of λ_1 and λ_2.■

At this moment, we arrive at evaluating the Uniformly Minimum Variance Unbiased Estimator (UMVUE) of conditional reliability measure for Exponential distributions, given by (2.5).

Theorem 1. Suppose that $X_1, \dots, X_n$ and $Y_1, \dots, Y_m$ are two independent random samples from Exponential distributions with parameters λ_1 and λ_2, respectively. The UMVUE of the quantity (2.5) is given by

$$\hat{R}^{|a,b}_{UMVUE} = \begin{cases} \frac{(m-1)\ g(b,S,m-2,n-1)}{Q(a,b,T,S,n-1,m-1)}, & a \le b < S \le T \\ \frac{(m-1)\ g(b,T,m-2,n-1)}{Q(a,b,T,S,n-1,m-1)}, & a \le b \le T \le S \\ 0, & a < T \le b < S \\ 1-\left(\frac{S-a}{S-b}\right)^{m-1} + \frac{(m-1)\ g(a,S,m-2,n-1)}{Q(a,b,T,S,n-1,m-1)}, & b < a \le S < T \\ 1 & b < S \le a < T \\ 1-\frac{(n-1)\ g(a,T,m-1,n-2)}{Q(a,b,T,S,n-1,m-1)}, & b < a < T \le S \\ not\ defined & b > S \ or \ T < a \end{cases} \quad (2.13)$$

where $T=\sum_{i=1}^{n} X_i$,. $S=\sum_{j=1}^{m} Y_j$, $g(u,v,p,q) = \int_u^v (S-z)^p (T-z)^q dz$ and $Q(a,b,T,S,n,m) = (T-a)^n (S-b)^m$.

Proof. It is well known that (T,S) is complete sufficient, so

$$\hat{R}^{|a,b}_{UMVUE} = P\big(X_1 > Y_1 \big| X_1 > a, Y_1 > b, (T,S)\big) = \frac{P\big(X_1>Y_1, X_1>a, Y_1>b \big| (T,S)\big)}{P\big(X_1>a, Y_1>b\big|(T,S)\big)}.$$

Calculating the above probabilities requires the joint conditional density function of (X_1, Y_1) w.r.t. (T,S). Independence of X-portions from Y-portions and some computation gives:

$$f_{X_1,Y_1|T,S}(x_1,y_1|t,s) = \frac{(n-1)(m-1)}{ts}\left(1-\frac{x_1}{t}\right)^{n-2}\left(1-\frac{y_1}{s}\right)^{m-2}, \ 0<x_1<t, 0<y_1<s.$$

Therefore,

$$P\big(X_1>a, Y_1>b \big| (T,S)\big) = \left(1-\frac{a}{T}\right)^{n-1}\left(1-\frac{b}{S}\right)^{m-1},$$

and

$$P\big(X_1>Y_1, X_1>a, Y_1>b\big|(T,S)\big) = \iint_{\boldsymbol{A}} f_{X_1,Y_1|T,S}(x,y|T,S)\,dx\,dy,$$

where $\boldsymbol{A} = \{(x,y) | x>y, a<x<T, b<y<S\}$. The set $\boldsymbol{A}$ varies by the changes in the order of a,b,S and T, as below:

$\boldsymbol{A}_{a\le b<S\le T} = \{(x,y) | y<x<T, b<y<S\}$,

$\boldsymbol{A}_{a\le b\le T\le S} = \{(x,y) | y<x<T, b<y<T\}$,

$\boldsymbol{A}_{b<S\le a<T} = \{(x,y) | a<x<T, b<y<S\}$,

$\boldsymbol{A}_{b<a\le S<T} = \boldsymbol{A}_{b<S\le a<T} - \{(x,y) | a<x<y, a<y<S\}$,

$\boldsymbol{A}_{b<a<T\le S} = \{(x,y) | a<x<T, b<y<x\}$,

$\boldsymbol{A}_{a<T\le b<S} = \{\emptyset\}$.

Computing $\iint_{\boldsymbol{A}} f_{X_1,Y_1|T,S}(x,y|T,S)\,dx\,dy$ for the above regions, gives the desired result. ■

Note that the estimator (2.13) is not a linear combination of two statistics T and S, so $Var(\hat{R}^{|a,b}_{UMVUE})$ does not agree with the Cramer-Rao's lower bound.

In the special case of $S=T$, (2.13) will reduce to the following simple form:

$$\hat{R}^{|a,b}_{UMVUE} = \begin{cases} \frac{m-1}{(m-1)+(n-1)}\left(\frac{S-b}{S-a}\right)^{n-1}, & a \le b \\ 1-\frac{n-1}{(m-1)+(n-1)}\left(\frac{S-a}{S-b}\right)^{m-1}, & b < a \end{cases} \quad (2.14)$$

Example 2. By Example 1, for Exponential distributed components, $R^{|b,b} = \frac{\lambda_2}{\lambda_1+\lambda_2}$, i.e., $R^{|b,b} = R$. In reliability literature this fact is called the memoryless property, which is the signature of Exponential distributions. Suppose that the random variable $X \sim \text{Gamma}(2,\beta_1)$ is independent of $Y \sim \text{Gamma}(2,\beta_2)$. By using the corresponding probability density and distribution functions $f_X(x) = \beta_1^2 x e^{-\beta_1 x}$, $\bar{F}_X(x) = 1-e^{-\beta_1 x}(\beta_1 x+1)$, $f_Y(y) = \beta_2^2 y e^{-\beta_2 y}$, $\bar{F}_Y(y) = 1-e^{-\beta_2 y}(\beta_2 y+1)$, and some calculation we arrive at:

$$R^{|b,b} = \frac{\beta_2^2}{(\beta_1+\beta_2)(\beta_1 b+1)(\beta_2 b+1)}\left(\beta_1 b^2 + b + \frac{2\beta_1 b+1}{\beta_1+\beta_2} + \frac{2\beta_1}{(\beta_1+\beta_2)^2}\right). \quad (2.15)$$

A simple computation gives $R = \frac{\beta_2^2}{(\beta_1+\beta_2)}\left(\frac{1}{\beta_1+\beta_2} + \frac{2\beta_1}{(\beta_1+\beta_2)^2}\right)$, which is completely different from (2.15), unless for $b = 0$. The later is trivial by definition $R^{|b,b} = R, b = 0$. ■

Nonparametric Estimation

A nonparametric method for estimating $R^{|a,b}$ has been given in this subsection. In some situations, distribution of data is unknown or computing $R^{|a,b}$ via Proposition 1 is very cumbersome and complicated. In comparison to MLE and UMVUE, a method of estimation which does not be related on the distribution of data, may reveal better results or at least be practical. By general formulas of conditional probability and independence we have

$$R^{|a,b} = \frac{P(X>Y, X>a, Y>b)}{P(X>a)P(Y>b)}, \quad (2.16)$$

whenever $P(X > a)P(Y > b) > 0$. For an event $\boldsymbol{D} \subseteq \boldsymbol{S}$, the nonparametric estimator of its probability is $\hat{P}(\boldsymbol{D}) = \frac{n(\boldsymbol{D})}{n(\boldsymbol{S})}$, where $n(\cdot)$ is the counting measure. Therefore we may estimate (2.16) by

$$\hat{R}_{NP}^{|a,b} = \frac{n(\boldsymbol{A})}{n(B_1)n(B_2)}, \quad (2.17)$$

where $\boldsymbol{A} = \{(x,y)|x > y, x > a, y > b\}$, $B_1 = \{x|x > a\}$ and $B_2 = \{y|y > b\}$.

Let $X_1, \dots, X_n \sim X$ and $Y_1, \dots, Y_m \sim Y$ are two independent random samples. In order to compute (2.17) we have

$$n(B_1) = \sum_{i=1}^{n} U(X_i - a), \quad n(B_2) = \sum_{j=1}^{m} U(Y_j - b), \quad (2.18)$$

and

$$n(\boldsymbol{A}) = \sum_{i=1}^{n}\sum_{j=1}^{m} U\big(X_i - Y_j\big)U(X_i - a)U(Y_j - b)), \quad (2.19)$$

where $U(t)$ is 1 for positive t and 0, otherwise.

More easy and rapid computation may performs, as we remove those samples X_i and Y_j for them $X_i \le a$ and $Y_j \le b$, some $i = 1, \dots, n$ and $j = 1, \dots, m$. The number of remaining samples of X_i and Y_j is $n(B_1)$ and $n(B_2)$, respectively. If the remaining samples have been denoted by $\{X_{ri}: i = 1, \dots, n(B_1)\}$ and $\{Y_{rj}: j = 1, \dots, n(B_2)\}$, then $n(\boldsymbol{A})$ may be obtained by the following formula:

$$n(\boldsymbol{A}) = \sum_{i=1}^{n(B_1)}\sum_{j=1}^{n(B_2)} U(X_{ri} - Y_{rj}). \quad (2.20)$$

In order to find some nonparametric confidence interval for $R^{|a,b}$, we are prepared now to compute the expected value and variance of $\hat{R}_{NP}^{|a,b}$. The main task of the following theorem, is to find these two quantities. For simplicity and without loss of generality, let $N = nm$.

Theorem 2. Let $p_1 = P(X > Y, X > a, Y > b)$, $p_2 = P(X > a, Y > b)$ and $\hat{R}_{NP}^{|a,b}$ is given by (2.17). Then,

(i) $\hat{R}_{NP}^{|a,b}$ is an unbiased estimator for $R^{|a,b}$.

(ii) For $0 \le p_1 \le p_2 \le 1$ and $p_2 \ne 0$, the variance of $\hat{R}_{NP}^{|a,b}$ is:

$$Var\big(\hat{R}_{NP}^{|a,b}\big) = (1-p_2)^N \frac{p_1}{p_2}\left(1 - \frac{p_1}{p_2}\right)\left(\sum_{i=1}^{N}\frac{1}{i(1-p_2)^i} - \frac{p_1}{p_2 - p_1}\right).$$

(iii) $\hat{R}_{NP}^{|a,b}$ is a consistent estimator for $R^{|a,b}$.

Proof. (i) First restate $\hat{R}_{NP}^{|a,b}$ in the following form:

$$\hat{R}_{NP}^{|a,b} = \frac{n(\boldsymbol{A})}{n(\boldsymbol{B})}, \quad (2.21)$$

where $\boldsymbol{B} = \{(x,y)|x > a, y > b\}$. Note that $\boldsymbol{A} \subseteq \boldsymbol{B}$, so $0 \le \frac{n(\boldsymbol{A})}{n(\boldsymbol{B})} \le 1$, which guarantees the existence of $E\big(\hat{R}_{NP}^{|a,b}\big)$ and $var\big(\hat{R}_{NP}^{|a,b}\big)$. Moreover, it can be shown that the joint probability mass function of $(n(\boldsymbol{A}), n(\boldsymbol{B}))$ is:

$$f_{n(\boldsymbol{A}),n(\boldsymbol{B})}(x,y) = \binom{N}{x, y-x} p_1^x (p_2 - p_1)^{y-x}(1 - p_2)^{N-y}; \quad (x,y) \in S_{n(\boldsymbol{A}),n(\boldsymbol{B})}, \quad (2.22)$$

where $S_{n(\boldsymbol{A}),n(\boldsymbol{B})} = \{(x,y)|x = 0, \dots, nm;\ x \le y \le N - x\}$. For a justification and explanation of (2.22) note that $\big(n(\boldsymbol{A}), n(\boldsymbol{B})\big) = (x,y)$ is equivalent to $\big(n(\boldsymbol{A}), n(\boldsymbol{B} - \boldsymbol{A}), n(\boldsymbol{B}^c)\big) = (x, y-x, N-y)$. We have used this equivalence, for demonstrating that $P(\boldsymbol{B} - \boldsymbol{A}) = P(\boldsymbol{B}) - P(\boldsymbol{A}) = p_2 - p_1$ which formerly was denoted by $P(X > a, Y > b, X < Y)$ and also for demonstrating the equivalence, $\binom{N}{x, y-x, N-y} = \binom{N}{x, y-x}$.

By using (2.21) and (2.22) the moment generating function of $\hat{R}_{NP}^{|a,b}$ is computed as:

$$M_{\hat{R}_{NP}^{|a,b}}(t) = \sum_{y=0}^{N}\sum_{x=0}^{y} e^{t\frac{x}{y}}\binom{N}{x, y-x} p_1^x (p_2 - p_1)^{y-x}(1-p_2)^{N-y}.$$

Furthermore, by $\binom{N}{x, y-x} = \binom{N}{y}\binom{y}{x}$ and the L'hopital's Rule at $t = 0$ we have:

$$M_{\hat{R}_{NP}^{|a,b}}(t) = (1-p_2)^N\left[t + \sum_{y=1}^{N}\binom{N}{y}\left(\left(\frac{p_2 - p_1}{1-p_2}\right)\left(1 + \frac{p_1 e^{\frac{t}{y}}}{p_2 - p_1}\right)\right)^y\right],$$

$t \in R$.

Regarding a statistical well-known theorem, since $M_{\hat{R}_{NP}^{|a,b}}(t)$ exists in a neighborhood of 0, all moments exist and $E(X^k)$ can be extracted by $\frac{\partial^k M_X(t)}{\partial t^k}|_{t=0}$. Straight computation shows that $\frac{\partial M_{\hat{R}_{NP}^{|a,b}}(t)}{\partial t}|_{t=0} = \frac{p_1}{p_2}$.

(ii) For computation of $Var(\hat{R}_{NP}^{|a,b})$ we have

$$\frac{\partial^2 M_{\hat{R}_{NP}^{|a,b}}(t)}{\partial t^2}|_{t=0} = (1-p_2)^N \frac{p_1}{p_2}\left(1-\frac{p_1}{p_2}\right)\left(\sum_{i=1}^{N}\frac{1}{i(1-p_2)^i} - \frac{p_1}{p_2-p_1}\right) + \frac{p_1^2}{p_2^2},$$

which clearly proves (ii).

(iii) By the fact that $\hat{R}_{NP}^{|a,b}$ is unbiased for $R^{|a,b}$, it is enough to show that $Var(\hat{R}_{NP}^{|a,b})$ tends to 0 as nm tends to infinity. By the results of part (ii), this variance may be rewritten in the following form:

$$Var(\hat{R}_{NP}^{|a,b}) = \frac{p_1}{p_2}\left(1-\frac{p_1}{p_2}\right)\left((1-p_2)^N \sum_{i=1}^{N}\frac{1}{i(1-p_2)^i} - (1-p_2)^N \frac{p_1}{p_2-p_1}\right).$$

Also, $(1-p_2)^N \to 0$, as $N \to \infty$. So, it is enough to show that $(1-p_2)^N \sum_{n=1}^{N}\frac{1}{n(1-p_2)^n} \to 0$, as $N \to \infty$. Fix $p_2 \in (0,1]$, and for given $\varepsilon > 0$ choose $n_\varepsilon \in N$ such that $\frac{1}{n_\varepsilon p_2} < \varepsilon$. Moreover,

$$(1-p_2)^N \sum_{n=1}^{N}\frac{1}{n(1-p_2)^n} = (1-p_2)^N \sum_{n=1}^{n_\varepsilon - 1}\frac{1}{n(1-p_2)^n} + (1-p_2)^N \sum_{n=n_\varepsilon}^{N}\frac{1}{n(1-p_2)^n}.$$

Clearly, first part tends to zero as N tends to infinity. For the second part we have

$$\sum_{n=n_\varepsilon}^{N}\frac{1}{n(1-p_2)^n} < \frac{1}{n_\varepsilon}\sum_{n=n_\varepsilon}^{N}\frac{1}{(1-p_2)^n} < \frac{1}{n_\varepsilon}\sum_{n=1}^{N}\frac{1}{(1-p_2)^n} = \frac{1}{n_\varepsilon p_2}((1-p_2)^{-N} - 1).$$ Therefore,

$$0 \le \lim_{N\to+\infty}(1-p_2)^N \sum_{n=n_\varepsilon}^{N}\frac{1}{n(1-p_2)^n} < \lim_{N\to+\infty}\frac{1}{n_\varepsilon p_2}(1-(1-p_2)^N) = \frac{1}{n_\varepsilon p_2} < \varepsilon.$$

So, for any given $\varepsilon > 0$, it concludes that $0 \le \lim_{N\to+\infty}(1-p_2)^N \sum_{n=1}^{N}\frac{1}{n(1-p_2)^n} < \varepsilon$, which completes the proof. ■

The proof of Theorem 2, does not depend on two sets $\boldsymbol{A}$ and $\boldsymbol{B}$. In fact a more general result holds which is the context of the following remark.

Remark 1. Let $X_1,\ldots,X_n \sim X$ and $Y_1,\ldots,Y_m \sim Y$ are two independent random samples. For any arbitrary set $\boldsymbol{D}$, define $P(\boldsymbol{D}) = P((X,Y)\in \boldsymbol{D})$ and $n(\boldsymbol{D}) = \sum_{i=1}^{n}\sum_{j=1}^{m} I_{\boldsymbol{D}}(X_i,Y_j)$ where $I_{\boldsymbol{E}}(\boldsymbol{t}) = \begin{cases}1 & \boldsymbol{t}\in \boldsymbol{E}\\ 0 & \boldsymbol{t}\notin \boldsymbol{E}\end{cases}$. In the other words, $n(\boldsymbol{D})$ is the number of pairs (X_i,Y_j) in $\boldsymbol{D}$. Then, for any two arbitrary sets $\boldsymbol{A}$ and $\boldsymbol{B}$ which $\boldsymbol{A}\subseteq \boldsymbol{B}$ and $P(\boldsymbol{B}) > 0$, we have $Var\left(\frac{n(\boldsymbol{A})}{n(\boldsymbol{B})}\right) = (1-P(\boldsymbol{B}))^{nm}\frac{P(\boldsymbol{A})}{P(\boldsymbol{B})}\left(1-\frac{P(\boldsymbol{A})}{P(\boldsymbol{B})}\right)\left(\sum_{i=1}^{nm}\frac{1}{i(1-P(\boldsymbol{B}))^i} - \frac{P(\boldsymbol{A})}{P(\boldsymbol{B})-P(\boldsymbol{A})}\right)$. Also, $\frac{n(\boldsymbol{A})}{n(\boldsymbol{B})}$ is an unbiased and consistent estimator for $\frac{P(\boldsymbol{A})}{P(\boldsymbol{B})}$.

Results

In this section, we perform a simulation study to assess the quality and efficiency of the introduced parameters and estimators. Moreover, this numerical process gives us a comparison among the MLE, UMVUE and nonparametric estimator for $R^{|a,b}$. Since the results for $a=b$ is the same as the unconditional case ($R^{|a,a} = R$), therefore the simulation has been only done for $a \neq b$. All results are mean of 5000 iteration and give numerical approximations for the corresponding expected values. For more clearification note that we have iterated our simulation 5000 times. In i^{th} iteration we have generated random samples with size n and m and $\hat{R}^{|a,b}{}_i$, $\hat{R}_{NP\ i}^{|a,b}$ and $\hat{R}_{UMVUE_i}^{|a,b}$ have computed. The values of $\hat{R}^{|a,b}$, $\hat{R}_{NP}^{|a,b}$ and $\hat{R}_{UMVUE}^{|a,b}$ demonstrated in Tables are mean of these 5000 computed estimates as follow:

$$\hat{R}^{|a,b} = \frac{\sum_{i=1}^{5000}\hat{R}^{|a,b}{}_i}{5000}$$

$$\hat{R}_{NP}^{|a,b} = \frac{\sum_{i=1}^{5000}\hat{R}_{NP\ i}^{|a,b}}{5000}$$

$$\hat{R}_{UMVUE}^{|a,b} = \frac{\sum_{i=1}^{5000}\hat{R}_{UMVUE_i}^{|a,b}}{5000}$$

Four different criteria have been used for investigating efficiency, effectiveness and potentialities of methods: i)-Bias, ii)-Mean Square Error (MSE), iii)-Coverage Probability (CP), and iv)-Length of Confidence Interval (LCI). In Table 2, these quantities for $\lambda_1 = 2$, $\lambda_2 = 1.6$, different values of n and m and for two groups of parameters a and b ($a=1, b=2$ and $a=3, b=1.4$) have been demonstrated. In order to view the effects of number of observations in performance of estimations, all parameters have been fixed, except n and m. For the case of $a=1$ and $b=2$, it has observed that larger sample sizes have more reliable results.

As has been excepted, the three criteria MSE, Bias

Table 2. MLE for $\lambda_1 = 2$ and $\lambda_2 = 1.6$, different sample sizes and two group of parameters a, b.

Quantities	Sample sizes								
n	5	10	10	20	25	35	50	100	200
m	4	4	15	20	30	35	45	100	200
	$a = 1, b = 2$ and $\lambda_1 = 2$, $\lambda_2 = 1.6$ which lead to $R^{\|a,b} = 0.0601$								
$\hat{R}^{\|a,b}$	0.0712	0.0689	0.066	0.064	0.0632	0.0622	0.0612	0.0608	0.0605
MSE	0.0048	0.0027	0.0024	0.0013	0.001	7.E-04	5.E-04	2.E-04	1.E-04
Bias	0.011	0.0087	0.0058	0.0038	0.0031	0.002	0.001	7.E-04	3.E-04
CP	0.7866	0.8572	0.8498	0.8854	0.896	0.912	0.92	0.9352	0.9426
LCI	0.2619	0.2	0.1868	0.1364	0.1217	0.1033	0.0865	0.0615	0.0435
	$a = 3, b = 1.4$ and $\lambda_1 = 2$, $\lambda_2 = 1.6$ which lead to $R^{\|a,b} = 0.9571$								
$\hat{R}^{\|a,b}$	0.9425	0.9433	0.9519	0.9537	0.9549	0.9548	0.9552	0.9561	0.9568
MSE	0.0045	0.0045	0.0012	9.E-04	6.E-04	5.E-04	4.E-04	2.E-04	1.E-04
Bias	-0.0146	-0.0138	-0.0052	-0.0034	-0.0022	-0.0023	-0.0018	-0.001	-3.E-04
CP	0.7492	0.7504	0.8618	0.8734	0.8922	0.9098	0.9124	0.937	0.9384
LCI	0.2495	0.246	0.1326	0.1131	0.0927	0.0863	0.0761	0.0512	0.0361

Table 3. Results for different values of a and b for $\lambda_1 = 1.5$, $\lambda_2 = 1.6, n = m = 30$.

Quantities	Results								
a	0.5	0.5	0.5	1.25	1.25	1.25	3	3	3
b	0.2	1	2.3	0.2	1	2.3	1	5	7
$R^{\|a,b}$	0.7289	0.217	0.0223	0.9348	0.7018	0.0829	0.9893	0.0157	5.E-04
$\hat{R}^{\|a,b}$	0.7301	0.2173	0.0249	0.9334	0.7015	0.085	0.9874	0.0177	8.E-04

and LCI are decreasing with respect to sample sizes n and m, and increasing for criterion CP. By giving attention to the small values of MSE and Bias, the performance of all estimation methods seems to be remarkable. However, for $a = 1, b = 2$, positive values of Bias denote a negligible overestimation in this case. The results for $a = 3, b = 1.4$, are the same as before, except for the sign of Bias which are negative, this shows an underestimation in this case.

Finally, Table 3 gives the results for different values of a and b, when $\lambda_1 = 1.5$, $\lambda_2 = 1.6$, $n = m = 30$. This table confirms that $R^{|a,b}$ and $\hat{R}^{|a,b}$ are both decreasing functions of the parameter b, while they are increasing functions of the parameter a, as claimed in Remark 1.

In the sequel of this section, we inspect for performance of UMVUE and $\hat{R}_{NP}^{|a,b}$ and will compare them with $\hat{R}^{|a,b}$. Since the variance of UMVUE does not have a closed form, for its computation we need to calculate the variance from some sample of UMVUEs. Regarding the formula of MSE, for small values of Bias the variance may be approximately equal to MSE (we have computed the variance of UMVUE by using this method for some cases when these calculations had been confirmed by simulation). This denotes that comparisons based on CP and LCI, does not involve further information than comparisons based on Bias and MSE. On the other hand, results of Tables 2 shows that the performance based on CP and LCI is similar to performance based on Bias and MSE. Therefore, comparison among UMVUE, MLE and $\hat{R}_{NP}^{|a,b}$ has been demonstrated only by using the Bias and MSE criteria.

This comparison has been demonstrated in Tables 4 and 5. Our findings show that $\hat{R}_{NP}^{|a,b}$ has the minimum Bias among three estimators, while this estimator has the worst performance w.r.t MSE criterion. The performance of MLE and UMVUE is approximately similar. Results indicate that for small sample sizes n and m, UMVUE has better performance whenever $R^{|a,b}$ is not close to 1. However, for small values of m and large n, UMVUE is more reliable in comparison with large m and small n.

Real Data Analysis

In this section, we will apply the conditional stress-strength parameter (2.1) to a pair of real data sets for illustrative purposes. For more details about the data in Table 6, see Xia et al. [12]. This data are well-known and have been used in several studies on stress-strength parameter. For instance, Saracoglu et al. [13] have used this data for estimating the stress-strength parameter of Exponential distribution under progressive type-II censoring. The data sets consist of breaking strengths of jute fiber at two different gauge lengths 10 mm and 20 mm. The gauges with 10 mm lengths have considered as strength X and gauges of 20 mm length as stress Y. The Kolmogorov-Smirnov's test illustrates an acceptable fitness to the Exponential distribution with parameter 0.0027 and 0.0029, respectively. The test's statistics and their corresponding p-values have been shown in Table

Table 4. Comparison among MLE, $\hat{R}_{NP}^{|a,b}$ and UMVUE for different sample sizes and $a = 0.5, b = 0.3$ and $\lambda_1 = 1$, $\lambda_2 = 0.25$ which lead to $R^{|a,b} = 0.239$.

Quantities	Results								
n	10	25	50	60	10	20	50	30	100
m	10	25	50	60	30	30	35	80	40
$\hat{R}^{\|a,b}$	0.2516	0.2438	0.2417	0.2408	0.2414	0.2436	0.2432	0.24	0.2431
$\hat{R}_{UMVUE}^{\|a,b}$	0.2317	0.2372	0.2379	0.2382	0.2319	0.2334	0.2372	0.2366	0.2372
$\hat{R}_{NP}^{\|a,b}$	0.239	0.24	0.2385	0.2387	0.2381	0.24	0.2401	0.2395	0.2395
$\text{MSE}(\hat{R}^{\|a,b})$	0.0066	0.0025	0.0012	0.001	0.0036	0.0025	0.0016	0.0012	0.0012
$\text{MSE}(\hat{R}_{UMVUE}^{\|a,b})$	0.0064	0.0025	0.0012	0.001	0.0038	0.0024	0.0015	0.0012	0.0012
$\text{MSE}(\hat{R}_{NP}^{\|a,b})$	0.016	0.0058	0.003	0.0023	0.0084	0.0056	0.004	0.0027	0.0031
$\text{Bias}(\hat{R}^{\|a,b})$	0.0126	0.0048	0.0027	0.0018	0.0024	0.0046	0.0042	0.001	0.0041
$\text{Bias}(\hat{R}_{UMVUE}^{\|a,b})$	-0.0074	-0.0018	-0.0011	-8.E-04	-0.0071	-0.0056	-0.0018	-0.0024	-0.0018
$\text{Bias}(\hat{R}_{NP}^{\|a,b})$	0	0.001	-5.E-04	-4.E-04	-0.001	0.001	0.0011	5.E-04	5.E-04

Table 5. Comparison among MLE, $\hat{R}_{NP}^{|a,b}$ and UMVUE for different sample sizes and $a = 1, b = 2$ and $\lambda_1 = 0.2$, $\lambda_2 = 0.25$ which lead to $R^{|a,b} = 0.4549$.

Quantities	Results								
n	10	15	25	50	60	5	40	80	70
m	10	15	25	50	60	10	60	20	60
$\hat{R}^{\|a,b}$	0.4487	0.4512	0.4534	0.4533	0.4529	0.4328	0.4525	0.4567	0.4527
$\hat{R}_{UMVUE}^{\|a,b}$	0.4593	0.4565	0.4578	0.4553	0.4569	0.4472	0.4541	0.4593	0.4558
$\hat{R}_{NP}^{\|a,b}$	0.4565	0.453	0.4522	0.4542	0.4549	0.4576	0.4549	0.4546	0.4553
$\text{MSE}(\hat{R}^{\|a,b})$	0.0121	0.0081	0.0049	0.0025	0.0022	0.02	0.0028	0.003	0.0028
$\text{MSE}(\hat{R}_{UMVUE}^{\|a,b})$	0.0142	0.0092	0.0052	0.0026	0.0022	0.0264	0.003	0.0033	0.0029
$\text{MSE}(\hat{R}_{NP}^{\|a,b})$	0.0261	0.0161	0.0097	0.0046	0.0039	0.0443	0.0052	0.0064	0.0051
$\text{Bias}(\hat{R}^{\|a,b})$	-0.0062	-0.0036	-0.0014	-0.0016	-0.0019	-0.022	-0.0023	0.0019	-0.0022
$\text{Bias}(\hat{R}_{UMVUE}^{\|a,b})$	0.0045	0.0017	0.003	5.E-04	0.0021	-0.0076	-7.E-04	0.0045	9.E-04
$\text{Bias}(\hat{R}_{NP}^{\|a,b})$	0.0016	-0.0019	-0.0026	-6.E-04	1.E-04	0.0027	1.E-04	-3.E-04	4.E-04

6. Both p-values are significantly larger than 0.05. Therefore, the null hypothesis that data have the Exponential distribution has not been rejected.

The estimation of conditional stress-strength quantity has been represented in Table 7. For different values of a and b, $R^{|a,b}$ has been estimated by three MLE, nonparametric and UMVUE methods.

Discussion

The conditional stress-strength parameter ($R^{|a,b}$) as an appropriate extension of the stress-strength parameter has been introduced. A general formula for computing $R^{|a,b}$ in the case of continuous random variable has been presented. Inferences concerning $R^{|a,b}$ have been

Table 6. The breaking strength of jute fiber.

10 mm			20 mm		
693.73	671.49	262.9	71.46	578.62	547.44
704.66	183.16	353.24	419.02	756.7	116.99
323.83	257.44	422.11	284.64	594.29	375.81
778.17	727.23	43.93	585.57	166.49	581.6
123.06	291.27	590.48	456.6	99.72	119.86
637.66	101.15	212.13	113.85	707.36	48.01
383.43	376.42	303.9	187.85	765.14	200.16
151.48	163.4	506.6	688.16	187.13	36.75
108.94	141.38	530.55	662.66	145.96	244.53
50.16	700.74	177.25	45.58	350.7	83.55
statistic	0.958		statistic	0.317	
p-value	0.727		p-value	0.666	

Table 7. Estimation of $R^{|a,b}$ by three methods for different values of a and b in jute fiber data.

$\boldsymbol{a}$	**4**	**2**	**10**	**1**	**15**	**50**	**200**	**250**
$\boldsymbol{b}$	2.25	5	1	10	14	100	195	300
$\hat{\boldsymbol{R}}^{|a,b}$	0.5202	0.5135	0.5303	0.5051	0.5191	0.4515	0.5247	0.4515
$\hat{\boldsymbol{R}}_{NP}^{|a,b}$	0.5489	0.5489	0.5489	0.5489	0.5489	0.4626	0.4647	0.3835
$\hat{\boldsymbol{R}}_{UMVUE}^{|a,b}$	0.4846	0.5139	0.4951	0.5058	0.4834	0.4536	0.4891	0.4528

accomplished for Exponential distribution and nonparametric case. Although, $R^{|a,b}$ has been computed for systems with Gamma distributed components, a detailed study in this case may be performed in another specified project.

Formerly, the stress-strength parameter have been studied in details for a wide range of distributions such as Weibull, Burr type, generalized Exponential, generalized Logistic and generalized failure rate distributions (See [3] for a comprehensive list). These distributions may have been considered for studying the conditional stress-strength parameter, as well. Moreover, in a Bayesian point of view, estimations for conditional stress-strength parameter may have been done for Exponential and other above mentioned distributions.

Bhattacharyya and Johnson [14], have introduced the multicomponent stress strength parameter for the situations that a system may have more than one component X, as:

$$R_{s,k} = P[\,at\ least\ s\ of\ X_1, \dots, X_k\ exceed\ Y\,]. \quad (5.1)$$

Eryilmaz [15], Pakdaman and Ahmadi [16], Rao et al. [17] and Dey et al. [18] have considered this parameter for different distributions. The extension of $R_{s,k}$ to some practical and suitable conditional version, may be another motivation and of interest.

Acknowledgements

The authors express their sincere thanks to editor and referees for their careful readings of the article and providing valuable comments which led to remarkable improvements of this article.

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