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Estimation of Individual Wave Solutions for the Nonlinear Dynamic Model in A Heterogeneous Quantum Magnetoplasma

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Abstract

In this paper, a quantum plasma system was considered to study a nonlinear turbulence model. The properties of nonlinear propagation of the solitary potential wave in two-dimensional heterogeneous quantum magnetoplasma were investigated using the quantum hydrodynamic model. It was assumed that in addition to the heterogeneity in this system, there is a magnetic field. If the collision frequency between the heavy particles (ions and neutral particles) is negligible, a nonlinear equation in two dimensions (2D), as well as the solutions of the plasma electrostatic potential, are obtained. For this purpose, the method of indeterminate coefficients, dimensionless conversion, travel wave conversion, etc. were used. A series of corresponding physical quantity properties was described by solving the individual solution of wave obtained for a quantum plasma system with a nonlinear model. The effects of the quantum Bohm potential on the single wave structure of the electrostatic potential are shown numerically in Figures 1 and 2. It was found that increasing the numerical density and amplitude of this wave decreases. The present study may play a significant role in understanding the properties of potential wave propagation in dense astrophysical plasma where quantum effects are useful.

Keywords: Inhomogeneous quantum magnetoplasma system; Nonlinear dynamic disturbed; Traveling wave transformation; Nonlinear partial differential equations.

Introduction

Classical plasmas (due to high temperatures and low density) are different from quantum plasmas (due to low temperatures and high density). By lowering the plasma temperature and wavelength comparable to the dimensions of the desired system, the plasma behaves like a Fermi gas and the behavior of charged particles is greatly affected by quantum effects [1-5].

The field of quantum plasmas has attracted a lot of attention in the plasma physics community due to its wide application. Numerous investigations in dense

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astrophysical environments (such as white dwarfs and neutron stars) [6], in dusty plasma [7, 8], in microelectronic devices [9], in plasma produced by intense laser beam [10], nonlinear optics [11, 12] etc., to understand the quantum effects on the linear and nonlinear wave propagation behavior in these systems. Much research work has been devoted to dusty magnetic plasma, as it plays an important role in the study of various fields of plasma science [13-15]. Jung et al. [16] investigated the quantum effects of electron-electron scattering in a high-temperature dense plasma. Kremp [17] discussed kinetic theory in a multiparticle system for time-dependent electromagnetic fields. Shukla [18] studied the behavior of the alpha wave in parallel with the magnetic field in quantum plasma [13-15].

Today, many studies have been done by researchers to solve nonlinear problems. Nonlinear equations play an important role in various fields of science including fluid dynamics, engineering, mathematics, and plasma physics [19-25]. Recently, many methods have been used to estimate the solution of nonlinear partial differential equations (NLPDE) [26-28]. Using the quantum hydrodynamic model (QHD) and the laws of conservation of motion, the classical plasma fluid model can be changed to the quantum state. These changes are caused by quantum statistics, including quantum diffraction, and an additional term called the Boehm potential has appeared in the equations of motion of charged particles. Propagation of linear and nonlinear waves in heterogeneous quantum plasma using the QHD model has been the subject of numerous researches. Manfredi et al. [1] investigated wave propagation in collisionless quantum plasma. Shukla and Stanflo [29] showed that there are new drifts in non-uniform quantum magnetoplasms, and the frequency of this drift wave led to a change in the electron Bohm potential. El Taibani and Wadati [30] studied the nonlinear quantum dust acoustic wave in a non-uniform quantum dust plasma and stated that the change of solitons is related to some plasma parameters. Massoud et al. [31] investigated the linear and nonlinear properties of quantum dust acoustic waves in a scattered quantum plasma and showed that quantum statistical terms and Bohm potential significantly change the length scale of these structures. The presence of transverse disturbance introduces an anisotropy in the system, which modifies the wave structure and stability of the system [32, 33]. Using the tanh-coth method [34] is a direct algebraic approach to create exact single-wave solutions in nonlinear partial differential equations. Sirendaoerji [35] estimated several new individual responses for the generalized mKdV equation based on the tanh hyperbolic function method. This method of complex algebraic calculations is of great importance in the search for exact solutions to arbitrary nonlinear equations [36-40].

In this paper, the nonlinear propagation characteristics of solitary waves in a heterogeneous quantum magnetoplasma system are investigated using the quantum plasma nonlinear dynamics model (QHD model). Here, using physico-mathematical methods and theories including the tanh-coth method, the quantum plasma system is investigated and the single solutions of wave in the nonlinear model are discussed as follows. In part 2, to investigate the nonlinear partial differential equation, we first used dimensionless conversion and then mobile wave conversion. In part 3, we obtain the individual solution of wave for the plasma potential in the electrostatic state. In the last part, in a conclusion, the method used was introduced as an efficient method. Therefore, by estimating single-wave solutions, the structural properties of other physical states can be predicted.

Mathematical Formulation

1. Set of nonlinear equations

A heterogeneous quantum magnetoplasm consisting of electrons, ions, and neutrals in the background was considered here. On the other hand, the magnetic field assumed equilibrium in the z-direction and density gradient and temperature in the x-direction. Regardless of the quantum statistical contribution and bohemian potential of ions and using the QHD model, the equations of electrons and ions will be as follows:

$$m_e n_e(\partial_t + V_e.\nabla)V_e = -en_e\left(E + \frac{1}{C}V_e \times B_\circ\right) - \nabla P_e + \frac{\hbar^2 n_e}{2m_e}\nabla(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}) \quad (1)$$

$$m_i n_i(\partial_t + V_i.\nabla)V_i = +en_i\left(E + \frac{1}{C}V_i \times B_\circ\right) - m_i n_i v_{in} V_i \quad (2)$$

In this relation $\boldsymbol{E} = -\boldsymbol{\nabla\phi}$ the electrostatic field is the electrostatic potential, $\boldsymbol{V_e}$, $\boldsymbol{P_e}$, $\boldsymbol{n_e}$, $\boldsymbol{m_e}$, and $\boldsymbol{e}$ are the Fermi velocity, pressure, density, mass, and charge of the electron, respectively (we have similarities for ions with index i, and v_{in} is the frequency of collisions between ions and neutrals) [41, 43].

Case 1. Using the parallel component of equation 1 (electron pressure placement $\boldsymbol{P_e = \frac{\hbar^2(3\pi^2)^{2/3}}{5m_e} n_e^{5/3}}$, expansion by the Taylor series and boundary conditions [44]) and the velocity component of equation 2 can be written:

$$\left(\frac{\widetilde{n_e}}{n_{e0}}\right)=\frac{3e\phi}{2KT_e}+\frac{3e^2\phi^2}{8K^2T_e^2}+\frac{9e\hbar^2}{16m_eK^2T_e^2}\nabla^2\phi \quad (3)$$

$$\left(\frac{\widetilde{n_i}}{n_{i0}}\right)=\frac{3e\phi}{2KT_e}+\frac{3e^2\phi^2}{8K^2T_e^2}+\frac{9e\hbar^2}{16m_eK^2T_e^2}\nabla^2\phi-\frac{1}{4\pi en_0}\nabla^2\phi \quad (4)$$

Case 2. Similarly for electron pressure placement $P_e=\frac{\hbar^2(3\pi^2)^{2/3}}{2m_e}n_e^{5/3}$ [25, 26]:

$$\left(\frac{\widetilde{n_e}}{n_{e0}}\right)=\frac{3e\phi}{5KT_e}+\frac{3e^2\phi^2}{50K^2T_e^2}+\frac{3}{5}(3\pi^2n_0)^{-2/3}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) \quad (5)$$

$$\left(\frac{\widetilde{n_i}}{n_{i0}}\right)=\frac{3e\phi}{5KT_e}+\frac{3e^2\phi^2}{50K^2T_e^2}-\underbrace{\frac{KT_e}{4\pi e^2n_0}}_{\lambda_e^2}\nabla^2\phi+\underbrace{\frac{3}{10}\left(\frac{5}{3}\right)(3\pi^2n_0)^{2/3}}_{H^2}\nabla^2\phi \quad (6)$$

Assuming the frequency of the quantum collision is small ($\nu_i\ll\frac{\omega_{in}}{K}\ll\nu_e$, and $F(0,\phi)=0$), the nonlinear dynamic model of the heterogeneous plasma is written as follows; for case 1.

$$\frac{3}{2}\frac{\partial^2\phi}{\partial t^2}+A_0\frac{\partial^2\phi^2}{\partial t^2}-\lambda_e^2\frac{\partial^4\phi}{\partial t^2\partial y^2}+H^2\frac{\partial^4\phi}{\partial t^2\partial y^2}-\varrho^2\frac{\partial^4\phi}{\partial t^2\partial y^2}$$

$$-\omega_{in}\varrho^2\frac{\partial^3\phi}{\partial t\partial y^2}+\frac{3}{2}\upsilon_*\frac{\partial^2\phi}{\partial t\partial y}-B_0\frac{\partial^2\phi^2}{\partial t\partial y}-C^2\frac{\partial^2\phi}{\partial z^2}=F(\varepsilon,\phi) \quad (7)$$

In here $A_0=\frac{-3e}{4KT_e}$, K the Boltzman constant, $T_e=\frac{\hbar^2}{2m_eK}(3\pi^2n_{e0})^{2/3}$ the temperature of electrons, $B_0=\frac{3(l_n-l_t)}{4B}$, B the magnetic field, $\lambda_e^2=(\frac{KT_e}{4\pi e^2n_0})$ the electron fermi wavelength, $H=\sqrt{\frac{9\hbar^2}{16m_eKT_e}}$ quantum parameter, $\upsilon_*=(\frac{-2cKT_e}{3eB})l_n$ the drift velocity, $C=\sqrt{\frac{KT_e}{m_i}}$ the quantum ion-acoustic speed, $\varrho=\sqrt{\frac{KT_e}{m\Omega^2}}$ ($\Omega=\frac{eB}{Cm_i}$ is the ion cyclotron frequency), the ion Larmor radius at electron temperature in quantum plasma [41, 43] For case 2.

$$\frac{3}{5}\frac{\partial^2\phi}{\partial t^2}+C_0\frac{\partial^2\phi^2}{\partial t^2}-\lambda_e^2\frac{\partial^4\phi}{\partial t^2\partial y^2}+H^2\frac{\partial^4\phi}{\partial t^2\partial y^2}-\varrho^2\frac{\partial^4\phi}{\partial t^2\partial y^2}-$$

$$\omega_{in}\varrho^2\frac{\partial^3\phi}{\partial t\partial y^2}+\frac{3}{5}\upsilon_*\frac{\partial^2\phi}{\partial t\partial y}-D_0\frac{\partial^2\phi^2}{\partial t\partial y}-C^2\frac{\partial^2\phi}{\partial z^2}=F(\varepsilon,\phi) \quad (8)$$

In here $C_0=\frac{-3e}{25KT_e}$, $D_0=\frac{3(l_n-l_t)}{10B}$ [43, 44].

2. Conversion without dimension

To provide a turbulent individual wave solution, the mathematical equations of the nonlinear partial differential (7 and 8) must be dimensionless, so assuming $\psi=e\phi/\mathrm{KT}$, $\Upsilon=y/\varrho$, $Z=z/\varrho$, and $\tau=\Omega t$ dimensionless transformations are obtained for these equations: In case 1.

$$\frac{\partial^2\psi}{\partial\tau^2}-p_1\frac{\partial^2\psi^2}{\partial\tau^2}-p_2\frac{\partial^4\psi}{\partial\tau^2\partial\Upsilon^2}-p_3\frac{\partial^3\psi}{\partial\tau\partial\Upsilon^2}-p_4\frac{\partial^2\psi}{\partial Z^2}-p_5\frac{\partial^2\psi}{\partial\tau\partial\Upsilon}-p_6\frac{\partial^2\psi^2}{\partial\tau\partial\Upsilon}=F(\varepsilon,\psi) \quad (9)$$

In here $p_1=\frac{1}{2}$, $p_2=\frac{2}{3}\frac{(\lambda_e^2+\varrho^2-H^2)}{\varrho^2}$, $p_3=\frac{2}{3}\omega_{in}$, $p_4=\frac{2}{3}(\frac{C}{\Omega\varrho})^2$, $p_5=\frac{\upsilon_*}{\Omega\varrho}$, and $p_6=\frac{KT_e}{2eBc}(l_n-l_t)$.

In case 2.

$$\frac{\partial^2\psi}{\partial\tau^2}-q_1\frac{\partial^2\psi^2}{\partial\tau^2}-q_2\frac{\partial^4\psi}{\partial\tau^2\partial\Upsilon^2}-q_3\frac{\partial^3\psi}{\partial\tau\partial\Upsilon^2}-q_4\frac{\partial^2\psi}{\partial Z^2}-q_5\frac{\partial^2\psi}{\partial\tau\partial\Upsilon}-q_6\frac{\partial^2\psi^2}{\partial\tau\partial\Upsilon}=F(\varepsilon,\psi) \quad (10)$$

In here $q_1=\frac{1}{5}$, $q_2=\frac{5}{3}\frac{(\lambda_e^2+\varrho^2-H^2)}{\varrho^2}$, $q_3=\frac{5}{3}\omega_{in}$, $q_4=\frac{5}{3}(\frac{C}{\Omega\varrho})^2$, $q_5=\frac{\upsilon_*}{\Omega\varrho}$, and $q_6=\frac{KT_e}{2eBc}(l_n-l_t)$(On the other hand $F(\varepsilon,\psi)=f\left(\varepsilon,\frac{KT_e}{e\psi}\right)$ and the7) dimensionless coefficients expressions p_i and q_i to the right of Equations 13 and 14 are omitted.

3. Individual wave solutions

In nonlinear dimensionless quantum plasma (Equations (9 and 10)), single wave solutions can be obtained using the following traveling wave converter.

$$\epsilon=\alpha\Upsilon+\beta Z-\gamma\tau \quad (11)$$

Where α, β are wavenumbers and γ is wave frequency. By placing relation (11) in equations (9 and 10) and assuming $\psi=\psi(\epsilon)$ for case 1 we have:

$$+2\gamma(p_6\alpha-p_1\gamma)\frac{d^2\psi^2}{d\epsilon^2}+(\gamma^2-p_4\beta^2+p_5\alpha\gamma)\frac{d^2\psi}{d\epsilon^2}-(\gamma^2-p_4\beta^2+p_5\alpha\gamma)\,\psi\frac{d^2\psi}{d\epsilon^2}+p_3\alpha^2\gamma\frac{d^3\psi}{d\epsilon^3}-p_2\alpha^2\gamma^2\frac{d^4\psi}{d\epsilon^4}=F(\varepsilon,\psi)$$

(8
) (12)

After sorting the above relation, we have

$$\frac{d^4\psi}{d\epsilon^4}+\chi_1\frac{d^3\psi}{d\epsilon^3}+\chi_2\frac{d^2\psi}{d\epsilon^2}+\chi_3\psi\frac{d^2\psi}{d\epsilon^2}+\chi_4\frac{d^2\psi^2}{d\epsilon^2}=Q(\varepsilon,\psi) \quad (13)$$

For case 1.

$$\chi_1 = -\frac{p_3}{p_2\gamma};\ \chi_2 = -\frac{\gamma^2 - p_4\beta^2 + p_5\alpha\gamma}{p_2\alpha^2\gamma^2};\ \chi_3 = +\frac{\gamma^2 - p_4\beta^2 + p_5\alpha\gamma}{p_2\alpha^2\gamma^2};\ \chi_4 = -\frac{2(p_6\alpha - p_1\gamma)}{p_2\alpha^2\gamma}$$
$$Q(\varepsilon,\psi) = -\frac{F(\varepsilon,\psi)}{p_2\alpha^2\gamma^2} \qquad (14)$$

For case 2. The results for case 2 are quite similar to the results of case 1, except that in Equation 14, wherever $\boldsymbol{p_i}$ was, $\boldsymbol{q_i}$ must be replaced.

Solitary of the Wave for the Plasma Potential in Electrostatic state

In the nonlinear traveling wave without dimension (Equation 13) we consider the perturbation solution as follows [42]:

$$\psi(\epsilon,\varepsilon) = \sum_{i=0}^{\infty} \psi_i(\epsilon)\,\varepsilon^i \qquad (15)$$

First, we place equation (15) in the traveling wave relation (Equation (13)), then we extend the nonlinear expression to the perturbation parameter ε. By combining expressions with the same power $\boldsymbol{\varepsilon^i}$ and setting their coefficients to zero, a nonlinear equation is obtained.

$$\frac{d^4\psi_0}{d\epsilon^4} + \chi_1\frac{d^3\psi_0}{d\epsilon^3} + \chi_2\frac{d^2\psi_0}{d\epsilon^2} + \chi_3\psi_0\frac{d^2\psi_0}{d\epsilon^2} + \chi_4\frac{d^2{\psi_0}^2}{d\epsilon^2} = 0 \qquad (16)$$

And quite similarly the linear equation is obtained:

$$\frac{d^4\psi_1}{d\epsilon^4} + \chi_1\frac{d^3\psi_1}{d\epsilon^3} + (\chi_2 + (2\chi_4 + \chi_3)\psi_0)\frac{d^2\psi_1}{d\epsilon^2} + 4\chi_4\frac{d\psi_0}{d\epsilon}\frac{d\psi_1}{d\epsilon} + (2\chi_4 + \chi_3\psi_0)\frac{d^2\psi_0}{d\epsilon^2}\psi_1 = Q(0,\psi_0) \qquad (17)$$

Assuming that Equation 16 has an individual wave solution, the answer to this equation can be written using the uncertain coefficient method of the hyperbolic function [42]. The general solution of the mKdV equation, using the auxiliary equation mapping method, will be as follows:

$$\psi(\epsilon) = \sum_{i=0}^{n} a_i\,G^i(\epsilon) + \sum_{i=-1}^{-n} b_{-i}\,G^i(\epsilon) + \sum_{i=2}^{n} c_i\,G^{i-2}(\epsilon)\frac{dG(\epsilon)}{d\epsilon} + \sum_{i=-1}^{-n} d_{-i}\,G^i(\epsilon)\,{dG(\epsilon)}/{d\epsilon} \qquad (18)$$

$$\psi(\epsilon) = a_0 + a_1G(\epsilon) + a_2G^2(\epsilon) + \frac{b_1}{G(\epsilon)} + \frac{b_2}{G^2(\epsilon)} + c_2\frac{dG(\epsilon)}{d\epsilon} + d_1\frac{dG(\epsilon)/d\epsilon}{G(\epsilon)} + d_2\frac{dG(\epsilon)/d\epsilon}{G^2(\epsilon)} + \cdots \qquad (19)$$

Which $\boldsymbol{a_i}$, $\boldsymbol{b_i}$, $\boldsymbol{c_i}$, and $\boldsymbol{d_i}$ are constants. In exchange for $\boldsymbol{G(\epsilon)} = \mathbf{tanh}(\boldsymbol{\epsilon})$, and $\boldsymbol{\epsilon = \alpha Y + \beta Z - \gamma\tau}$ (14) results in the following derivatives:

$$dG(\epsilon)/d\epsilon = 1 - \tanh^2(\epsilon);$$
$$d^2G(\epsilon)/d\epsilon^2 = -2\tanh(\epsilon)\left(1 - \tanh^2(\epsilon)\right) + \left(1 - \tanh^2(\epsilon)\right)^2;\ \ldots \qquad (20)$$

Using the finite expansion, the expression tanh-coth can be written as follows, in which M is obtained using the HBM method. After determining the fixed parameters, an analytical solution $\boldsymbol{\psi(\epsilon,t)}$ is obtained (these solutions can be traveling wave, soliton, or periodic solutions [36].

$$\psi_0(\epsilon) = \sum_{k=0}^{M} a_k\,G^k(\epsilon) + \sum_{k=1}^{M} b_k\,G^{-k}(\epsilon) \qquad (21)$$

$$\psi_0(\epsilon) = a_0 + a_1\tanh(\epsilon) + a_2\tanh^2(\epsilon) + \frac{b_1}{\tanh(\epsilon)} + \frac{b_2}{\tanh^2(\epsilon)} \qquad (22)$$

$\boldsymbol{a_i}$, $\boldsymbol{b_i}$ are indeterminate constants that can be obtained by placing Equation 22 in Equation 16 and then combine the coefficients $\mathbf{tanh}(\boldsymbol{\epsilon})$) By placing $\boldsymbol{a_i}$ and $\boldsymbol{b_i}$ in relation (22) and by assuming one of them to be zero ($\boldsymbol{b_1 = 0}$; $\boldsymbol{b_2 = 0}$);

$$a_0 = -\frac{12\alpha^2p_3^2p_2 + 10p_5\alpha p_3p_2}{2p_3(10p_2\alpha p_6 - p_1p_3)} - \frac{-100p_4p_2^2\beta^2 + p_3^2}{2p_3(10p_2\alpha p_6 - p_1p_3)};\ a_1 = \frac{12p_2\alpha^2p_3}{10p_2\alpha p_6 - p_1p_3};$$
$$a_2 = \frac{6p_2\alpha^2p_3}{10p_2\alpha p_6 - p_1p_3};\ \gamma = \frac{p_3}{10p_2} \qquad (23)$$

(17)

$$\psi_{10} = \frac{12\alpha^2p_3^2p_2 + 10p_5\alpha p_3p_2}{2p_3(10p_2\alpha p_6 - p_1p_3)} - \frac{-100p_4p_2^2\beta^2 + p_3^2}{2p_3(10p_2\alpha p_6 - p_1p_3)} - \frac{6p_2\alpha^2p_3(\tanh(\epsilon))^2}{p_1p_3 - 10p_2\alpha p_6} - \frac{12p_2\alpha^2p_3\tanh(\epsilon)}{p_1p_3 - 10p_2\alpha p_6} \qquad (24)$$

2) By assuming $\boldsymbol{a_1 = 0}$; $\boldsymbol{a_2 = 0}$ we have:

$$a_0 = -\frac{12\alpha^2p_3^2p_2 + 10p_5\alpha p_3p_2}{2p_3(10p_2\alpha p_6 - p_1p_3)} - \frac{-100p_4p_2^2\beta^2 + p_3^2}{2p_3(10p_2\alpha p_6 - p_1p_3)};\ b_1 = \frac{12p_2\alpha^2p_3}{10p_2\alpha p_6 - p_1p_3}$$
$$b_2 = \frac{6p_2\alpha^2p_3}{10p_2\alpha p_6 - p_1p_3};\ \gamma = \frac{p_3}{10p_2} \qquad (25)$$

$$\psi_{20} = -\frac{12\alpha^2p_3^2p_2 + 10p_5\alpha p_3p_2}{2p_3(10p_2\alpha p_6 - p_1p_3)} + \frac{-100p_4p_2^2\beta^2 + p_3^2}{2p_3(10p_2\alpha p_6 - p_1p_3)} + \frac{12p_2\alpha^2p_3}{(10p_2\alpha p_6 - p_1p_3)\tanh(\epsilon)} + \frac{6p_2\alpha^2p_3}{(10p_2\alpha p_6 - p_1p_3)(\tanh(\epsilon))^2} \qquad (26)$$

Then two single wave solutions are obtained as follows.

$$\psi_{10}(\epsilon) = I_0 + I_1(2-\tanh(\epsilon))\tanh(\epsilon) \tag{27}$$

$$\psi_{20}(\epsilon) = I_0 + I_1\frac{(2\tanh(\epsilon)+1)}{\tanh^2(\epsilon)} \tag{28}$$

So that $\boldsymbol{I_0}$ and $\boldsymbol{I_1}$ are constants as follows:

$$I_0 = -\frac{6\alpha^2 p_2 p_3 + 5\alpha p_2 p_5 - 50(\beta^2 p_2^2 p_4)/p_3 + 0.5 p_3}{10 p_2 \alpha p_6 - p_1 p_3} \tag{29}$$

$$I_1 = \frac{6\alpha^2 p_2 p_3}{10 p_2 \alpha p_6 - p_1 p_3} \tag{30}$$

Where $\boldsymbol{\psi_0}$ is a defined function (obtained from relations (27) and (28), See figures 1 and 2). In this study, a graphical analysis was presented by drawing the curve of zero wavelength ($\boldsymbol{\psi_{10}(\epsilon), and\ \psi_{20}(\epsilon)}$) in the electrostatic potential of quantum plasma against different parameters affecting the wave. In Figures 1 and 2, the change of wave potential with density is shown. From the comparison of these figures, it can be seen that the two single zero waves ($\boldsymbol{\psi_{10}(\epsilon), and\ \psi_{20}(\epsilon)}$) are completely different in strength due to the choice of different physical parameters (on a dimensionless scale). According to Figures, it is clear that increasing the number density decreases the amplitude. Since the quantum Boehm potential includes the density, then the density change indirectly indicates the change of the wave potential (quantum Boehm potential).

Note: The second term that includes the $\boldsymbol{\tanh(\epsilon)}$ term in the equation is responsible for the shock-like structure because it destroys the balance between dispersion and nonlinearity.

Using the perturbation theory ([42] and Equation (16)), $\boldsymbol{\psi_1}$ and $\boldsymbol{\psi_2}$ can be obtained from the first approximate solutions of a single wave.

$$\psi_1(\epsilon,\varepsilon) = I_0 + I_1(2-\tanh(\epsilon))\tanh(\epsilon) + \varepsilon\psi_{21}(\epsilon) + O(\varepsilon^2); \quad 0<\varepsilon\ll 1 \tag{31}$$

$$\psi_2(\epsilon,\varepsilon) = I_0 + I_1\frac{(2\tanh(\epsilon)+1)}{\tanh^2(\epsilon)} + \varepsilon\psi_{11}(\epsilon) + O(\varepsilon^2); \quad 0<\varepsilon\ll 1 \tag{32}$$

Next, and similarly, the nth approximate asymptotic solutions ($\boldsymbol{\psi_{1n}(\epsilon,\varepsilon)}$ and $\boldsymbol{\psi_{2n}(\epsilon,\varepsilon)}$) can be obtained. Using transformer (17), the nth solution for two waves individually of dimensionless nonlinear dynamic electrostatic potential was obtained:

$$\Phi_{1n}(\epsilon,\varepsilon) = I_0 + I_1(2-\tanh(\epsilon))\tanh(\epsilon) + \sum_{i=1}^{n}\psi_{1i}(\epsilon)\,\varepsilon^i + O(\varepsilon^{n+1}); \quad 0<\varepsilon\ll 1 \tag{33}$$

$$\Phi_{2n}(\epsilon,\varepsilon) = I_0 + I_1\frac{(2\tanh(\epsilon)+1)}{\tanh^2(\epsilon)} + \sum_{i=1}^{n}\psi_{1i}(\epsilon)\,\varepsilon^i + O(\varepsilon^{n+1}); \quad 0<\varepsilon\ll 1 \tag{34}$$

Results

One of the most important applications of non-uniform quantum dynamic equations of plasma system is to study the effect of systems potential solutions, and analysis of potential amplitude, and explosion wave density. Since the plasma turbulence system arises from a natural phenomenon, for the nonlinear solo study of more models, approximations must be used. In this paper, a quantum plasma system was considered to study a nonlinear turbulence model. The properties of nonlinear propagation of the solitary potential wave in two-dimensional heterogeneous quantum magnetoplasma were investigated using the quantum hydrodynamic model. It was assumed that in addition to the heterogeneity in this system, there is a magnetic field. If the collision frequency between the heavy particles (ions and neutral particles) is negligible, a nonlinear equation in two dimensions (2D), as well as the solutions of the plasma electrostatic potential, is obtained. For this purpose, the use of approximate methods including the hyperbolic function method, indeterminate coefficient method, and perturbation theory was used as a very efficient method. A series of corresponding physical quantity properties was described by solving the individual solution of wave obtained for a quantum plasma system with a nonlinear model. The effects of the quantum Bohm potential on the single wave structure of the electrostatic potential are shown numerically in Figures 1 and 2. It was found that increasing the numerical density and amplitude of this wave decreases. The present study may play a significant role in understanding the properties of potential wave propagation in dense astrophysical plasma where quantum effects are useful. Therefore, single wave solutions for other physical states can be investigated using the solution method proposed here (decomposition operation) in the future.

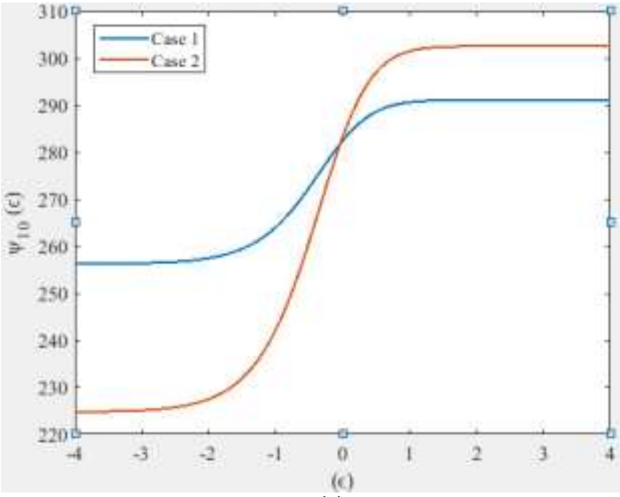

Figure 1. Image for the zero-wavelength ($\psi_{10}(\epsilon)$) diagram of quantum plasma electrostatic potential for case 1 and case 2, per $n_0 = 1.2 \times 10^{26} cm^{-3}$, $B = 2 \times 10^{12} G$, $T_e = 1.03 \times 10^7 K$, $\varrho = 2.15 \times 10^{-9} cm$, $n_0 = 2.02 \times 10^{-9} cm$, $H = 6.95 \times 10^{-10} cm$, $C = 2.06 \times 10^7 cm/s$, $\omega_{in} = 9.58 \times 10^{14} Hz$, and $\upsilon_* = 1.65 \times 10^6 cm/s$.

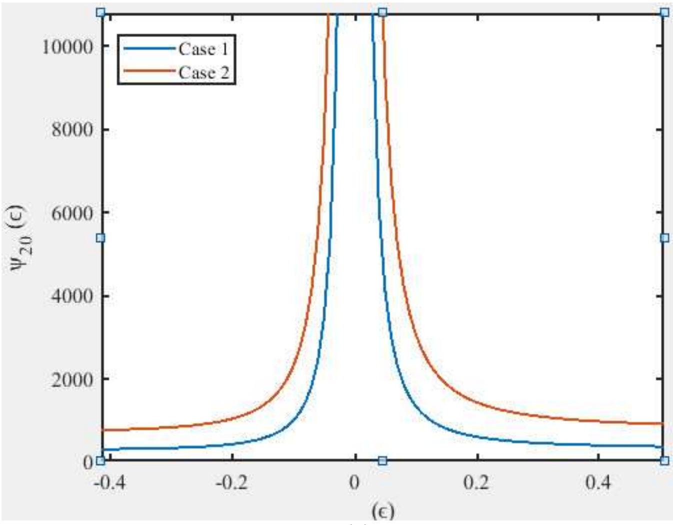

Figure 2. Image for the zero-wavelength ($\psi_{20}(\epsilon)$) diagram of quantum plasma electrostatic potential for case 1 and case 2.

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