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Qutrit Teleportation and Entanglement Evolved by the One-Axis Counter-Twisting Hamiltonian under the Intrinsic Decoherence

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Abstract

In this paper, we study the entanglement and quantum teleportation of a two-qutrit state evolved under one-axis counter-twisting Hamiltonian with the intrinsic decoherence effects. The entanglement and fidelity are analyzed as a function of decoherence rate, Hamiltonian coefficient, and magnetic field. It has been seen that the system is constantly entangled. Both the decoherence rate and the Hamiltonian coefficient have a negative correlation with the entanglement and fidelity. The faithfulness and negativity are efficiently optimized by the magnetic fields. We deduced that we can acquire some best fidelity for the system when it is maximally entangled.

Keywords Entanglement; Teleportation; Intrinsic decoherence; One-axis counter-twisting Hamiltonian.

Introduction

The quantum entanglement is a fundamental concept in many quantum information processes, such as quantum teleportation, dense coding, quantum communication and quantum cryptography (1-6). Much attentions was paid to the one-axis counter-twisting Hamiltonian because of creating entanglement and quantum correlations used in spin systems (7-9).

The one-axis counter-twisting Hamiltonian have experiment realization in Bose-Einstein condensate(10), and optical lattice (11). Recently, spin chains were used in quantum teleportation (12-14). Due to their superior capacity and security than qubit states, qutrits, or three level atoms, are employed for quantum teleportation (15). On the other hand, many approaches have been used to address the consequences of noises or decoherences in the real world (16). Milburn for the intrinsic decoherence proposed a scheme to modify the ordinary Schrodinger equation (17-19). The spin chains as the channel for teleportation under decoherence change from pure state to mixed states, and it affects the fidelity (20). The effect of intrinsic decoherence on entanglement dynamics and quantum teleportation was studied in several works (21-24).

This study aims to investigate the time evolution of entanglement for two-qutrit state under one-axis counter-twisting (OAT) Hamiltonian (7) with intrinsic decoherence effects (17), and the teleportation via this channel. The paper is set up as follows. Section 1 introduces the system's Hamiltonian in the presence and absence of a magnetic field and obtains the density matrix of the system with intrinsic decoherence. We examine the quantum system's entanglement in section 2. We examine the teleportation via this channel in section 3. Finally, section 5 is devoted to discussion.

Materials and Methods

1. The Hamiltonian of the system

Kitagawa and Ueda introduced two nonlinear interaction so called one-axis, and two-axes counter-twisting Hamiltonian to generate correlations and spin

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squeezing (7). We consider the interaction of two three-level atoms by OAT Hamiltonian. This Hamiltonian for the system is given by:

$$H = \chi J_x^2 \qquad (1)$$

where $J_x = \sum_{i=1}^{N} J_{ix}$ is the x component of the collective angular momentum for spin 1.

Moreover, we study OAT interaction in the presence of magnetic fields with the other Hamiltonian is written as

$$H_1 = \chi J_x^2 + B J_z \qquad (2)$$

where B is the magnetic field in the Z direction and χ is Hamiltonian coefficient .

The master equation describing the intrinsic decoherence under Markovian approximations is given by (17):

$$\frac{d\rho(t)}{dt} = -i\left[H,\rho(t)\right] - \frac{\Gamma}{2}\left[H,\left[H,\rho(t)\right]\right] \qquad (3)$$

where Γ is the intrinsic decoherence rate. The formal solution of above master equation can be expressed as (19):

$$\rho(t) = \sum_{k=0}^{\infty} \frac{(\Gamma t)^k}{k!} M^k \rho(0) M^{\dagger k} \qquad (4)$$

where $\rho(0)$ is the density operator of the initial state and M^k is defined by:

$$M^k = H^k e^{-iHt} e^{-\frac{\Gamma t}{2}H^2} \qquad (5)$$

Based on Eq (4), it is easy to show that under intrinsic decoherence, the dynamics of the density operator for above-mentioned system which is initially in the state $\rho(0)$ is given by (25):

$$\hat{\rho}(t) = \sum_{m,n} \exp\left[\frac{-t\Gamma}{2}(E_m - E_n)^2 - i(E_m - E_n)t\right] \times \langle\psi_m|\hat{\rho}(0)|\psi_n\rangle |\psi_m\rangle\langle\psi_n| \qquad (6)$$

where the system is initially in the state $\rho(0)$; $E_i\,(i = m,n)$ and $\psi_i\,(i = m,n)$ are eigenvalues, and corresponding eigenvectors of $\hat{H}$ respectively.

A qutrit system is described by the basis $|0\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $|1\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}$, $|2\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$ in three dimensional Hilbert space.

We assume that two-qutrit system is initially prepared at one qutrit is in $|1\rangle$ state and the other qutrit is in $|0\rangle$ state as the follows:

$$|\psi_0\rangle = |1\rangle \otimes |0\rangle \qquad (7)$$

Therefore, $\rho(0)$ can be written as:

$$\rho_0 = |1,0\rangle\langle 1,0|$$

$$\rho_0 = \begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \qquad (8)$$

The Hamiltonian (1) can be written in the following matrix form:

$$H = -\hbar \begin{pmatrix} \frac{\chi}{2}&0&\frac{\chi}{4}&0&\frac{\chi}{2}&0&\frac{\chi}{4}&0&0\\ 0&\frac{3\chi}{4}&0&\frac{\chi}{2}&0&\frac{\chi}{2}&0&\frac{\chi}{4}&0\\ \frac{\chi}{4}&0&\frac{\chi}{2}&0&\frac{\chi}{2}&0&0&0&\frac{\chi}{4}\\ 0&\frac{\chi}{2}&0&\frac{3\chi}{4}&0&\frac{\chi}{4}&0&\frac{\chi}{2}&0\\ \frac{\chi}{2}&0&\frac{\chi}{2}&0&\chi&0&\frac{\chi}{2}&0&\frac{\chi}{2}\\ 0&\frac{\chi}{2}&0&\frac{\chi}{4}&0&\frac{3\chi}{4}&0&\frac{\chi}{2}&0\\ \frac{\chi}{4}&0&0&0&\frac{\chi}{2}&0&\frac{\chi}{2}&0&\frac{\chi}{4}\\ 0&\frac{\chi}{4}&0&\frac{\chi}{2}&0&\frac{\chi}{2}&0&\frac{3\chi}{4}&0\\ 0&0&\frac{\chi}{4}&0&\frac{\chi}{2}&0&\frac{\chi}{4}&0&\frac{\chi}{2} \end{pmatrix} \qquad (9)$$

where the basis is $\{|0,0\rangle,|0,1\rangle,|0,2\rangle,|1,0\rangle,|1,1\rangle,|1,2\rangle,|2,0\rangle,|2,1\rangle,|2,2\rangle\}$, and Hamiltonian affect these states as in the following:

Using Eq. (9) and Eq. (8) and Eq. (6), we can obtain $\rho(t)$ of the whole system. This matrix is too complicated to be given here.

2. *Entanglement of the system*

Negativity is a straightforward, calculable metric. Vidal et al. have demonstrated that negativity is an entanglement monotone and a suitable entanglement measure (26). It is also demonstrated that the negativity can be measured experimentally using a quantum circuit devoid of noise (27). The negativity $N(\rho)$ for an arbitrary bipartite system is defined as in the following (26):

$$N(\rho) = \frac{\left\| \rho^{T_A} \right\| - 1}{2} \quad (10)$$

where $\left\| \rho^{T_A} \right\| = \mathrm{Tr}\sqrt{\left(\rho^{T_A}\right)^{\dagger} \rho^{T_A}}$ denotes the trace norm of ρ^{T_A}. The ρ^{T_A} is the partial transpose ρ with respect to the subsystem A. The negativity $N(\rho)$ is equivalent to the absolute value of the sum of negative eigenvalues of ρ^{T_A}.

To discuss the entanglement dynamics in the system discussed in section 2, we can calculate the negativity by Eqs. (6) and (10). We have presented the negativity of two atoms as a function of time for various Γ in Figure 1. The plot shows that the entanglement

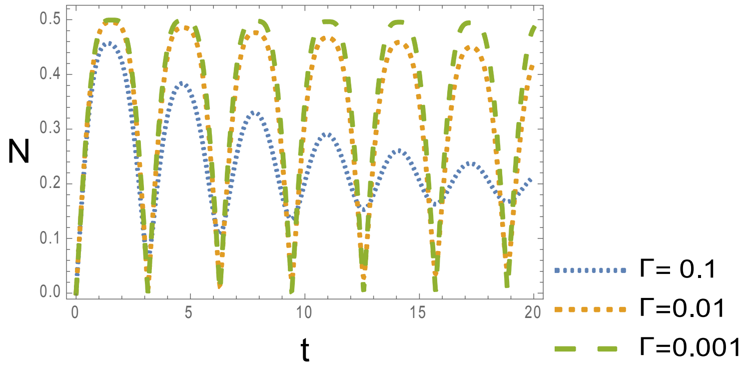

Figure 1. Negativity as a function of time for various values of Γ assuming $\chi = 1$.

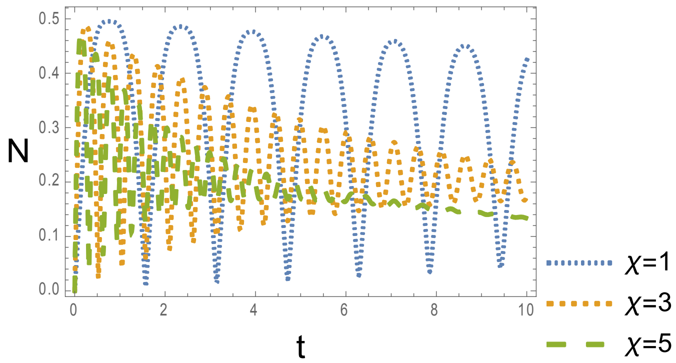

Figure 2. Negativity as a function of time for various values χ and assuming Γ=0.01.

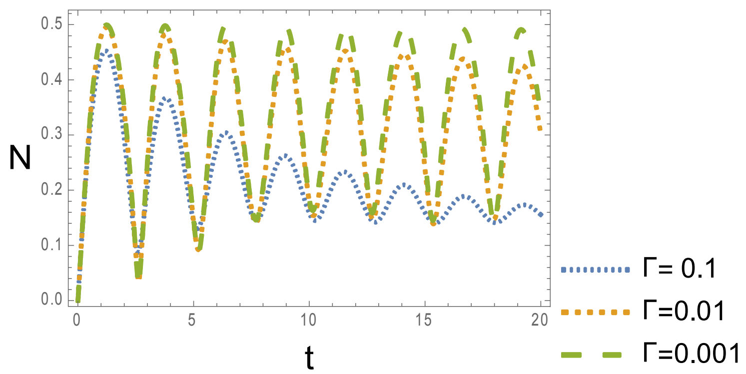

Figure 3. Negativity as a function of time for various values of Γ for $\chi = 1$, B=1..

evolves periodically with decreasing amplitude by passing time. It is observed that the amplitude of entanglement is a decreasing function of Γ but its frequency is an independent function of Γ. In Figure 2, we showed the time evolution of negativity for different values χ. It is deduced that the negativity is a decreasing function of χ. Moreover, for given values of χ the entanglement is initially an oscillating function of time, but it tends to be a constant value with the passage of time. Also, the frequency is an increasing function of χ and the amplitude is a decreasing function of χ. So, negativity goes to the constant more rapidly for higher values of χ

We examine the effects of the magnetic field on the system using the Hamiltonian H_1. In Figure 3, the negativity was plotted as a function of Γ and t for Hamiltonian H_1. It is observed that the negativity is decreased with the increase of Γ. Besides, the entanglement is an oscillating function of time for small value of Γ. In the comparison with Figure 1, It is found that the magnetic field optimize the entanglement in the average.

In Figure 4, we have presented the negativity as a function of time and different values of B assuming $\Gamma = 0.01$. In the comparison with Figure 2, It is observed that the amplitude of the entanglement is increasing function of B. On the other hand, the frequency of entanglement is decreasing function of magnetic fields and over time the negativity tends to be a constant value.

3. Teleportation

Bose (28) investigates teleportation utilizing spin chains. According to this standard teleportation, Alice placed her qutrit in the i'th position of a spin chain, while Bob placed his qutrit in the j'th position. Then, Alice will use this entangled spin chain as Bob's channel to transmit an unknown state. After some operations on qutrit 1 and 3, Alice measures the two qutrits in her possession, and then sends this information to Bob. Depending on Alice's classical message, Bob performs some unitary operations on his half of the EPR pair and he can inform of the original state.

Alice is initially given the pure state $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$, where $|\psi_{in}\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle)$.

$$\rho_{in} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{11}$$

The aim of Alice is to send this state eq (11) via the channel with the density matrix in equation (6). The output state is given by (28)

$$\rho_{out} = \sum_{j=0}^{8} Tr[E^j \rho(t)]\{\Gamma^j \rho_{in} \Gamma^j\} \tag{12}$$

where Γ^j $(j = 1,..,8)$ refers to Gell-Mann matrices and Γ^0 is the identity matrix for qutrits systems. Also, E^j's are the density matrices of maximum entangled qutrit states as

$\phi^0 = (1/\sqrt{3})(|2,0\rangle + |1,1\rangle + |0,2\rangle)$,

$\phi^1 = (1/\sqrt{3})(|1,0\rangle + |0,1\rangle + |2,2\rangle)$,

$\phi^2 = (1/\sqrt{3})(|0,0\rangle + |2,1\rangle + |1,2\rangle)$,

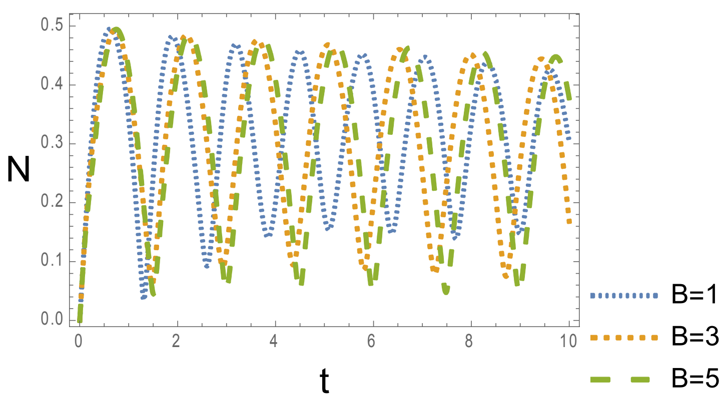

Figure 2. Negativity as a function of time for various values B assuming Γ=0.01.

$$\phi^3 = \left(1/\sqrt{3}\right)\left(\left|2,0\right\rangle + e^{\frac{2\pi i}{3}}\left|1,1\right\rangle + e^{-\frac{2\pi i}{3}}\left|0,2\right\rangle\right),$$

$$\phi^4 = \left(1/\sqrt{3}\right)\left(\left|1,0\right\rangle + e^{\frac{2\pi i}{3}}\left|0,1\right\rangle + e^{-\frac{2\pi i}{3}}\left|2,2\right\rangle\right),$$

$$\phi^5 = \left(1/\sqrt{3}\right)\left(\left|0,0\right\rangle + e^{\frac{2\pi i}{3}}\left|2,1\right\rangle + e^{-\frac{2\pi i}{3}}\left|1,2\right\rangle\right),$$

$$\phi^6 = \left(1/\sqrt{3}\right)\left(\left|2,0\right\rangle + e^{-\frac{2\pi i}{3}}\left|1,1\right\rangle + e^{\frac{2\pi i}{3}}\left|0,2\right\rangle\right),$$

$$\phi^7 = \left(1/\sqrt{3}\right)\left(\left|1,0\right\rangle + e^{-\frac{2\pi i}{3}}\left|0,1\right\rangle + e^{\frac{2\pi i}{3}}\left|2,2\right\rangle\right),$$

$$\phi^8 = \left(1/\sqrt{3}\right)\left(\left|0,0\right\rangle + e^{-\frac{2\pi i}{3}}\left|2,1\right\rangle + e^{\frac{2\pi i}{3}}\left|1,2\right\rangle\right).$$

The quality of the teleported state will be measured in terms of fidelity. The fidelity of two states is given by (29-30)

$$F(\rho_{\text{in}},\rho_{\text{out}}) = \{Tr[\sqrt{\sqrt{\rho_{\text{in}}}\,\rho_{\text{out}}\sqrt{\rho_{\text{in}}}}\,]\}^2 \quad (14)$$

where ρ_{in} is the input state of the channel and ρ_{out} is the output state of the channel.

We have plotted the fidelity as function of time for various values of Γ in Figure 5. It is observed that the fidelity is decreased by passing time. The amplitude of fluctuation is decreasing function of Γ.

We have plotted the fidelity for various values of χ in Figure 6. It is observed that the frequency of the fluctuation of fidelity is increasing function of χ. The fidelity is decreased rapidly for smaller χ in average.

We have studied the effects of magnetic fields by using Hamiltonian H_1. We have plotted this fidelity as function of time in the presence of magnetic field in Figure 7. It is observed that the magnetic fields effectively optimize the fidelity. For smaller value of χ, the magnetic fields cause to increase the fidelity

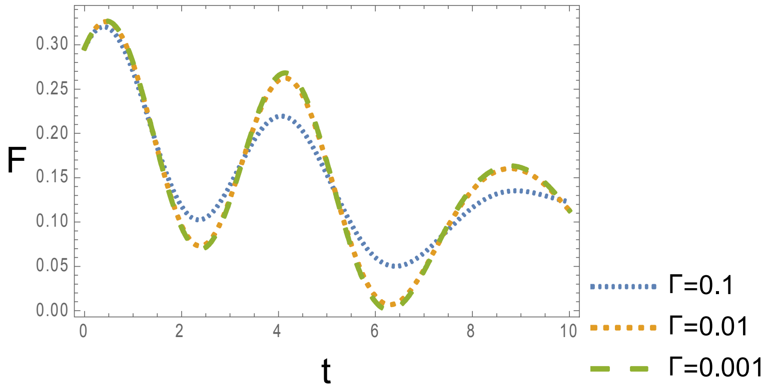

Figure 5. Fidelity as a function of time for various values of Γ for $\chi = 1$.

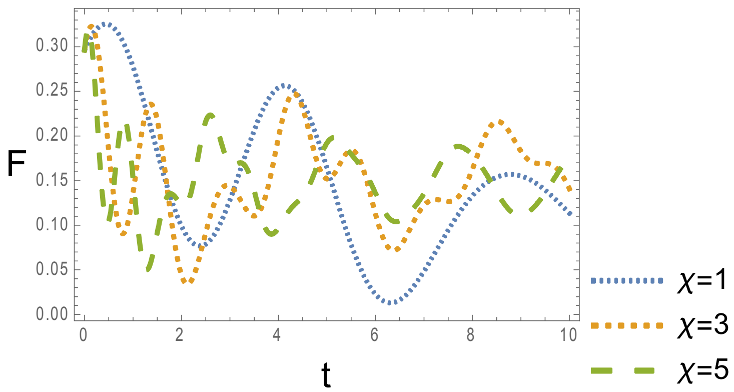

Figure 6. Fidelity as a function of time for various values of χ for $\Gamma = 0.02$.

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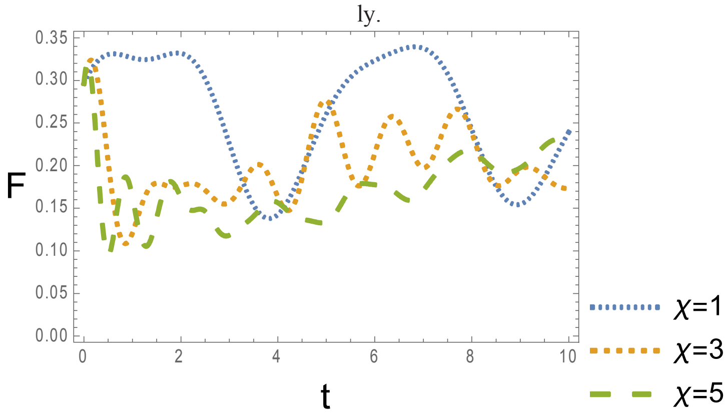

Figure 7. Fidelity as a function of time for various values of χ for $\Gamma = 0.02$, B=1.

effectively.

twisting Hamiltonian can be maximally entangled.

Results and Discussion

In both the presence and absence of a magnetic field, we investigated the dynamics of entanglement in a two-qutrit state governed by the OAT Hamiltonian with the inherent decoherence effects. The analysis of the results showed that the entanglement is decreasing function of both the intrinsic decoherence parameter Γ and the Hamiltonian coefficient χ. Besides, the entanglement is an increasing function of magnetic field and the entanglement goes to constant value with the passage of time.

We studied the teleportation via this entangled spin chain. We observed that the fidelity is decreasing function of parameter Γ. The reason is that the coherence of the system can be preserved in longer time with smaller decoherence parameter therefore the entanglement and fidelity can be enhanced by decreasing the coherence parameter. Additionally, the magnetic field has an impact on fidelity, making this effect more advantageous at lower parameter values. Because the magnetic field makes the spins rotate, they may be correlated.It is observed that with the increase or decrease of the entanglement, the fidelity of channel correspondingly goes up or down. Nevertheless, We deduce that one-axis counter-twisting Hamiltonian generates the entangled states, and it is useful for teleportation in virtue of these qutrit channels.

Considering the application of the teleportation in the transmitting of encoding quantum information and the security of the transfer of high-fidelity information, by manuplating the parameters appropriately, we can have a high-fidelity channel for the transfer of desired states. Furthermore, this channel using one-axis counter-

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