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Adomian Decomposition Approach to Solve the Quantum Mesoscopic Nonlinear LC-circuits with Charge Discreetness

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Abstract

The quantum Hamitonian of a nonlinear mesoscopic LC-circuit include a nonlinear inductor and a linear capacitor with charge discreetness is introduced. An analytical function for quantum persistent current due to the magnetic flux of such nonlinear electrical circuit is obtained by Adomian Decomposition Method (ADM). The nonlinear quantum ring is introduced and the quantum persistent current and the eigenvalues energy are found analytically. It is shown by numerical solution that the persistent current and eigenvalues energy are periodic functions of the magnetic flux with the fundamental parameter ($\hbar/e$), which is a pure quantum characteristic. In this paper, we proved for the first time that in a mesoscopic nonlinear quantum ring, similar a linear quantum ring, there is a quantized and periodic persistent current in terms of magnetic flux, which will attract attention in experience and technology.

Keywords: Charge discreteness; Duffing equation; Analytic solution; Mesoscopic circuits; Nonlinear inductor.

Introduction

The Adomian Decomposition Method (ADM) is powerful mathematical tool for solving wide class of nonlinear differential equations (1-8). ADM is a useful method for expressing approximate analytical solutions that provide effective algorithms in engineering and science. ADM has been widely used in solving partial differential equations that can be used in various phenomena in engineering and physics.

Over the past few decades, the ADM has been successful in extracting analytical solutions of linear and nonlinear differential equations by providing solutions based on the convergent power series. Unlike traditional numerical methods, this method (ADM) does not require linearization and discretization of variables and therefore does not face the problem of computational rounding errors. And this method has been used in various problems such as nanotubes (7), to solve the simple Harmonic quantum oscillator (2), for nonlinear wave-like equations with variable coefficient (5), fluid dynamics analysis (9) and others. Primary electrical LC-circuit modeling, including capacitors and nonlinear inductors, was first studied in (1915) by Biermanns (10). The nonlinear inductor has a ferromagnetic core that up to now, were not taken into in the quantum mesoscopic electrical LC-circuit particularly in the charge discrete conditions(11,12). To be taken into account the quantum effects, the transport dimension of electric devices in the electric circuits must reaches to coherence length. which in this article is in the mesoscopic order, 10 to 1000 nm, and in this article the electric charge is a discrete quantity

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and therefore quantum effects are observed in the electric circuit. For this propose, in the references (13-15), by reducing the inductor dimensions, the inductance magnitude has reached the level of nanohenry.

Research on electronic, thermodynamic, quantum and etc. properties quantum nano and mesoscale structures, such as quantum circuits and quantum rings have made a lot of progress in recent years (16-18). So far, a lot of research has been done on the dynamics of mesoscopic quantum electrical linear LC-circuits and nonlinear LC-circuits (with nonlinear capacitor) based on Li and Chen theory (11, 12, 19-21). Nonlinear inductors have not yet been considered in discrete charge quantum mesoscopic electrical circuits. In this paper, we consider an electrical nonlinear circuit include a linear capacitor and a nonlinear inductor under the action external field based on Li and chen thory and dynamics behavior is investigated by apply the ADM. We calculate the persistent current function, which is an observable quantity in mesoscopic electrical circuits (22), in a nonlinear ring and conclude that it varies periodically with the flux (ϕ), because the study of quantum rings is a very interesting subject in mesoscopic physics, in which the Aharonov-Bohm effect can be observed, and they can be used in various fields such as quantum interferometers (23, 24).

The quantum Hamiltonian

The electrical nonlinear circuit include a linear capacitor and a nonlinear inductor under the action external field is showed in the Figure 1.

This nonlinear circuit was classically studied many years ago by Biermenns (10), Hayashi (25) and Ueda (26). The nonlinear relationship between the current and the magnetic flux can be represented by several different functions. The nonlinear inductor is an inductor with a ferromagnetic core, whose nonlinearity is expressed by relation (1), which is written as a simple representation of the power series in references (10) and (25)

$$i=\sum_{k=1}^{\infty} a_k \phi^k \qquad (1)$$

$$i = a_1\phi + a_3\phi^3 + a_5\phi^5 + \ldots$$

Where a_1, a_3, a_5, ... are constants characterizing the core i.e., they depend on the type of the inductor. The governing differential equation of the Figure 1 using the equation (1) up to the third order is given by

$$\frac{d^2\phi}{dt^2}+\frac{1}{L_1C}\phi+\frac{1}{L_3C}\phi^3=0 \qquad (2)$$

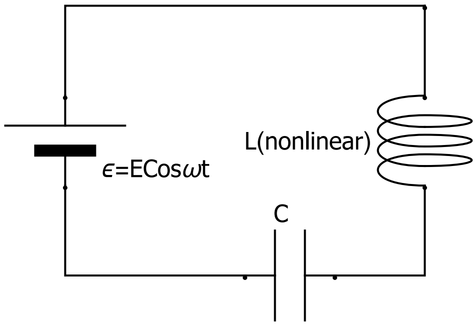

Figure 1. A nonlinear electric circuit including a linear capacitor and a nonlinear inductor under the influence of an external field.

Where $\frac{1}{L_n}=\frac{a_n}{N}$ n=1,3 and the natural frequency of this system is $\omega_n=\frac{1}{\sqrt{L_1C}}$. The classical Hamiltonian of equation (2) (nonlinear circuit Figure 1) is given as follows

$$H=\frac{p^2}{2L_1}+\frac{p^4}{4L_3}+\frac{q^2}{2C} \qquad (3)$$

With define the parameters $L_1 = L(a_1=\frac{1}{L})$ and $\frac{1}{4L_3}=\lambda$ (9), in the Hamiltonian (3), we have

$$H=\frac{p^2}{2L}+\lambda p^4+\frac{q^2}{2C} \qquad (4)$$

where L is induction of the inductor (linear coefficient), λ is nonlinear coefficient and C is a linear coefficient (capacity). Based on the Louisell work (27), the quantum Hamiltonian of this nonlinear circuit can be written as follows

$$\hat{H}=\frac{1}{2L}\hat{p}^2+\lambda\hat{p}^4+\frac{\hat{q}^2}{2C} \qquad (5)$$

And $[\hat{q},\hat{p}]=i\hbar$, where, the $\hat{q}$ and $\hat{p}$ operators are continuous variables of the electric charge and current. In order to take the discreteness of electronic charge, Li and Chen proposed a quantum theory for mesoscopic circuits (11,12,28,29). In this theory, the discreteness of an electric charge is considered by a self-adjoint operator $\hat{q}$ that has a discrete spectrum as

$$\hat{q}|n\rangle = nq_e|n\rangle \qquad (6)$$

where $|n\rangle$ is the eigenvectors of operator $\hat{q}$ in the charge space, and $n \in Z$ (set of integers) and $q_e = 1.602\times10^{-19}C$ the elementary charge electric. Also, a minimum shift operator $\hat{Q}$ is introduced by Li and Chen as

$$\hat{Q} = e^{iq_e \hat{\varphi}/\hbar} \tag{7}$$

where $\hat{\varphi}$ is the magnetic flux operator threading the inductor and satisfy the usual commutation rule

$$[\hat{q},\hat{\varphi}] = i\hbar \tag{8}$$

The operators $\hat{Q}$, $\hat{Q}^\dagger$ and $\hat{q}$ fulfill the following commutation relations

$$\begin{aligned}&[\hat{q},\hat{Q}] = -q_e\hat{Q}\\ &[\hat{q},\hat{Q}^\dagger] = q_e\hat{Q}^\dagger\\ &[Q,Q^\dagger] = 0\end{aligned} \tag{9}$$

Based on theory Li and Chen's, the momentum operator will be obtained

$$\hat{P} = \frac{\hbar}{2iq_e}(\hat{Q}-\hat{Q}^\dagger) \tag{10}$$

And the quantum Hamiltonian of the mesoscopic LC-circuit with electrical charge discreteness is given as follows

$$\hat{H} = -\frac{\hbar^2}{2Lq^2_e}(\hat{Q}+\hat{Q}^\dagger-2)+V(\hat{q}) \tag{11}$$

However, when the inductor exhibits nonlinear behavior, the inductance is not a constant and is a function of the current, the quantum mesoscopic Hamiltonian (11) circuit including the linear capacitor and the nonlinear inductor in the charge discreteness conditions can be written as follows

$$\hat{H} = -\frac{\hbar^2}{2q_e{}^2L'}(\hat{Q}+\hat{Q}^\dagger-2)+\lambda(\frac{\hbar}{q_e})^4(\hat{Q}^2+\hat{Q}^{\dagger2}-2)+\frac{\hat{q}^2}{2C} \tag{12}$$

where

$$\frac{1}{L'} = \frac{1}{L}+8\lambda\frac{\hbar^2}{q_e{}^2} \tag{13}$$

The persistent current of nonlinear LC-circuit

In order to study quantum dynamics the quantum mesoscopic Hamiltonian (11), let us calculate the time evolution of the operators $\hat{Q}$, $\hat{Q}^\dagger$ using the Heisenberg equations of motion as follows

$$\begin{aligned}&\dot{\hat{Q}}(t) = -i\frac{q_e}{\hbar}(\frac{\hat{q}}{C}+\frac{q_e}{2C})\hat{Q},\\ &\dot{\hat{Q}}^\dagger(t) = i\frac{q_e}{\hbar}\hat{Q}^\dagger(\frac{\hat{q}}{C}+\frac{q_e}{2C})\end{aligned} \tag{14}$$

Equations (14) are not exactly solvable due to the nonlinear sentences $\hat{q}\hat{Q}$ and $\hat{Q}^\dagger\hat{q}$,therefore, the operator $\hat{Q}(t)$ is written as follows

$$\hat{Q}(t) = \hat{A}(t)\exp(-\frac{iq_e}{\hbar}\int_0^t\frac{q_e}{2C}dt') \tag{15}$$

And as a result

$$\hat{Q}(t) = \hat{A}(t)\exp(-\frac{iq_e{}^2}{2\hbar C}t) \tag{16}$$

Substituting this equation in the equations (14), we have

$$\dot{\hat{A}}(t) = -\frac{iq_e}{\hbar C}\hat{q}(t)\hat{A}(t) \tag{17}$$

The above differential equation with the following condition

$$\hat{A}(t,t_0=0) = \hat{A}(0) \tag{18}$$

is equivalent to the following integral equation (28,29)

$$\hat{A}(t) = \hat{A}(0)-\frac{iq_e}{\hbar C}\int_0^t\hat{q}(t')\hat{A}(t')dt' \tag{19}$$

Approximate solution of this equation up to the first order approximation with assuming

$$\hat{A}(t') = \hat{A}(0) \tag{20}$$

is given

$$\hat{A}(t) = (1-\frac{iq_e}{\hbar C}\int_0^t\hat{q}(t')dt')\hat{A}(0) \tag{21}$$

Now, we should determine the charge operator $\hat{q}(t)$. For tis propose, the operator $\dot{\hat{q}}$ is written as follows

$$\begin{aligned}\hat{I}(t) &= \frac{1}{i}[\hat{q},\hat{H}]\\ &= \frac{\hbar}{2iq_eL'}(\hat{Q}(t)-\hat{Q}^\dagger(t))-\frac{2\lambda}{i}(\frac{\hbar}{q_e})^3(\hat{Q}^2(t)-\hat{Q}^{\dagger2}(t))\end{aligned} \tag{22}$$

And so, we have

$$\ddot{\hat{q}}(t) = \frac{\hbar}{2iLq_e}(\dot{\hat{Q}}-\dot{\hat{Q}}^\dagger)+8\lambda(\frac{\hbar}{q_e})^3(\hat{Q}\dot{\hat{Q}}-\hat{Q}^\dagger\dot{\hat{Q}}^\dagger) \tag{23}$$

Substituting the equation (14) into the equations (23), we have

$$\ddot{\hat{q}}(t) = -\frac{\hbar}{2iLq_e}\left(-\frac{iq_e}{\hbar}\left(\frac{\hat{q}}{C}+\frac{q_e}{2C}\right)(\hat{Q}-\hat{Q}^\dagger)\right)+8\lambda\left(\frac{\hbar}{q_e}\right)^3\left(-\frac{iq_e}{\hbar}\left(\frac{\hat{q}}{C}+\frac{q_e}{2C}\right)\right)(\hat{Q}^2-\hat{Q}^{\dagger 2}) \tag{24}$$

Equation (24) can not be solvled according to conventional mesoscopic approaches and we use another method to calculate the operator $\hat{q}$. Using the Heisenberg equations of motion, the time evolution $\hat{p}$ operator is given by

$$\dot{\hat{P}} = \frac{1}{i\hbar}[\hat{P},\hat{H}] = -\frac{\hat{q}}{C} \tag{25}$$

By deriving from equation (25) and substituting equation (22), we have

$$\ddot{\hat{P}} = -\frac{1}{C}\frac{\hbar}{2iq_eL'}(\hat{Q}-\hat{Q}^\dagger)+\frac{2\lambda}{iC}\left(\frac{\hbar}{q_e}\right)^3(\hat{Q}^2-\hat{Q}^{\dagger 2}) \tag{26}$$

For a finite value of q_e, namely 1.6×10^{-19}, we are in the continuous charge regime, so it is expected for us to choose q_e as a perturbative parameter and expand the shift operators $\hat{Q}$ and $\hat{Q}^\dagger$ in the powers of q_e. Here, we expand these operators up to the order q_e^3, i.e.

$$\hat{Q} = e^{iq_e\hat{p}/\hbar} = 1+(iq_e\frac{\hat{p}}{\hbar})+\frac{1}{2!}(iq_e\frac{\hat{p}}{\hbar})^2+\frac{1}{3!}(iq_e\frac{\hat{p}}{\hbar})^3+... \tag{27}$$

$$\hat{Q}^\dagger = e^{-iq_e\hat{p}/\hbar} = 1-(iq_e\frac{\hat{p}}{\hbar})+\frac{1}{2!}(iq_e\frac{\hat{p}}{\hbar})^2-\frac{1}{3!}(iq_e\frac{\hat{p}}{\hbar})^3+... \tag{28}$$

Substituting equation (27) into (26), after some calculations, we will find the quantum Duffing equation as follows

$$\ddot{\hat{P}} = -\frac{1}{LC}\hat{P}-\frac{4\lambda}{C}\hat{P}^3 \tag{29}$$

In this article, we use the Adomian decomposition method Because this method, based on technique of decomposition, makes approximate or even exact solutions with suitable initial conditions for nonlinearity with easy application. ADM, is used as an effective, simple, and popular method, to solve various linear, nonlinear, and integral equations. Therefore, in the following of paper, we apply ADM to solve the equation (29) and obtain the persistent current on a quantum mesoscopic electrical nonlinear LC-circuit with charge discreteness.

After applying the Adomin decomposition method and converting the series solution into a continuous equation calculate $\hat{p}(t)$ operator, whose steps are given in the appendix, we can Now from the equation (25) the time-dependent operator $\hat{q}$ is given by

$$\hat{q}(t) = \hat{q}(0)\sin(\omega' t) \tag{30}$$

where $\omega+\sqrt{\frac{\lambda p_0^2}{C}}=\omega'$. By Substituting equation (30) into (21) we have

$$\hat{A}(t) = (1-i\frac{q_e}{\hbar C}\int_0^t \hat{q}(0)\sin(\omega' t')dt')\hat{A}(0) \tag{31}$$

Using the equation (31), we can find the operators $\hat{Q}$ and $\hat{Q}^\dagger$ as

$$\hat{Q}(t) = (1+i\frac{q_e}{\hbar C}\frac{\cos(\omega' t)}{\omega'}\hat{q}(0))\exp(-i\frac{q_e^2}{2\hbar C}t)\hat{Q}(0)$$
$$\hat{Q}^\dagger(t) = \hat{Q}^\dagger(0)(1-i\frac{q_e}{\hbar C}\frac{\cos(\omega' t)}{\omega'}\hat{q}(0))\exp(i\frac{q_e^2}{2\hbar C}t) \tag{32}$$

Now, using these equations, from the equation (22), we can obtain the current operator i.e.

$$\begin{aligned}\hat{I}(t) =& \frac{\hbar}{q_eL'}\sin(\frac{q_e}{\hbar}(\hat{p}-\frac{q_e}{2C}t))+\frac{1}{L'C}\frac{\cos\omega' t}{\omega'}\cos(\frac{q_e}{\hbar}(\hat{p}-\frac{q_e}{2C}t))\hat{q}(0)\\ &-4\lambda(\frac{\hbar}{q_e})^3\{\sin(\frac{2q_e}{\hbar}(\hat{p}-\frac{q_e}{2C}t))(1-\frac{\cos^2\omega' t}{\omega'}(\frac{q_e}{2\hbar})^2\hat{q}^2(0))\\ &+\frac{2q_e}{\hbar C}\hat{q}(0)\frac{\cos\omega' t}{\omega'}\cos(\frac{2q_e}{\hbar}(\hat{p}-\frac{q_e}{2C}t))\}\end{aligned} \tag{33}$$

And as a result, the persistent current is obtained

$$\begin{aligned}I_\varphi(t) =& \langle I_\varphi|\hat{I}(t)|I_\varphi\rangle\\ I_\varphi(t) =& \frac{\hbar}{q_eL'}\sin(\frac{q_e}{\hbar}(\varphi-\frac{q_e}{2C}t))-4\lambda(\frac{\hbar}{q_e})^3\{\sin(\frac{2q_e}{\hbar}(\varphi-\frac{q_e}{2C}t))\}\end{aligned} \tag{34}$$

In the above equation, $\langle I_\varphi|\hat{q}(0)|I_\varphi\rangle = 0$ is used and according to equation (13), we have

$$I_\varphi(t) = \frac{\hbar}{q_eL}\sin(\frac{q_e}{\hbar}(\varphi-\frac{q_e}{2C}t))+8\lambda(\frac{\hbar}{q_e})^3\sin(\frac{q_e}{\hbar}(\varphi-\frac{q_e}{2C}t))(1-\cos(\frac{q_e}{\hbar}(\varphi-\frac{q_e}{2C}t))) \tag{35}$$

Equation (35) indicates that the persistent current on quantum mesoscopic LC circuit include a linear capacitor and a nonlinear inductor with charge discreteness. Our formulation presented a method from a new point of view to the quantum for nonlinear circuits. In order to determine the degree of validity of Equation (35) at the limiting $q_e\to0$, we have

$$I_\varphi = \frac{\varphi}{L}+4\lambda\varphi^3$$

Nonlinear quantum ring

In this section, we consider the nonlinear electrical circuit that shown in the Figure 1, only including only one nonlinear inductor. Since the core material of the nonlinear inductor is usually made of ferromagnetic material. Thus, we have a quantum mesoscopic ring with ferromagnetic core that is a natural pure nonlinear L design.

$$\hat{H} = -\frac{\hbar^2}{2{q_e}^2 L'}(\hat{Q}+\hat{Q}^{\dagger}-2)+\lambda\left(\frac{\hbar}{q_e}\right)^4(\hat{Q}^2+\hat{Q}^{\dagger 2}-2) \quad (36)$$

The eigenvalues energy are

$$\begin{aligned} E &= \frac{1}{L'}\left(\frac{\hbar}{q_e}\right)^2\left(1-\cos\left(\frac{q_e\varphi}{\hbar}\right)\right)-2\lambda\left(\frac{\hbar}{q_e}\right)^4\left(1-\cos\left(\frac{2q_e\varphi}{h}\right)\right) \\ &= \left(\frac{\hbar}{q_e}\right)^2\left(\frac{1}{L}+8\lambda\left(\frac{\hbar}{q_e}\right)^2\right)\left(1-\cos\left(\frac{q_e\varphi}{\hbar}\right)\right)-2\lambda\left(\frac{\hbar}{q_e}\right)^4\left(1-\cos\left(\frac{2q_e\varphi}{h}\right)\right) \end{aligned} \quad (37)$$

The diagram of the energy spectrum of the nonlinear quantum ring is drawn in terms of φ in the Figure 2 for various λ values . In which, it can see that the energy has increased with the increase of the nonlinear coefficient of the inductor. The energy level in the linear ring ($\lambda = 0$) is lower than the nonlinear ring. The energy of a mesoscopic nonlinear quantum pure L design cannot be larger than $(\frac{2\hbar}{q_e^2})$ that it differing with the usual nonlinear classical pure L design. From the analytical relationship (37) and diagram (2), it can be concluded that the limit state approximately is $(\frac{q}{\hbar})^2$, the amount of energy will not exceed the limit

The persistent current (the eigenvalues of the electric current operator) of a non-linear mesoscopic ring is abstained as follows

$$I_{\varphi}(t)=\frac{\hbar}{q_e L}\sin\left(\frac{q_e}{\hbar}\varphi\right)+8\lambda\left(\frac{\hbar}{q_e}\right)^3\sin\left(\frac{q_e}{\hbar}\varphi\right)\left(1-\cos\left(\frac{q_e}{\hbar}\varphi\right)\right) \quad (38)$$

Based on, when in the equation (35) $C \to \infty$,the persistent current of the nonlinear quantum ring in the charge discreetness conditions is given by equation (38).

By plotting the persistence current curve as a function of φ in Figure 3, it was observed that for different values of λ, the periodicity of the persistence current curve in a non-linear quantum ring is remained constant

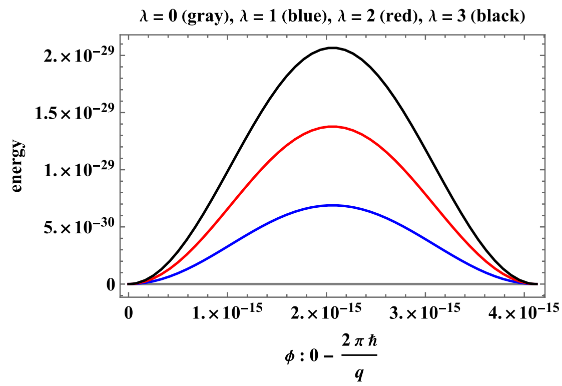

Figure 2. Energy of a nonlinear quantum ring vs φ for various λ values.
$\hbar = 1.05\times10^{-34} J.s, q_e = 1.6\times10^{-19} C, L = 2\times10^{-10} H$

which indicates the quantum reality of this system. Also, the persistent current value fluctuates between two values of $\left(\frac{\hbar}{Lq_e}\right)$ and $-\left(\frac{\hbar}{Lq_e}\right)$, up to the order of magnitude of λ equal nearly to $(\frac{q}{\hbar})^2$. As it is known, in the non-linear mesoscopic ring, despite the non-linearity of the inductor the persistent current is still a periodic function of the magnetic flux with the fundamental parameter ($\hbar/e$). This is a pure quantum characteristic

Results

Also, today, the desire to use portable electronic devices is increasing, so due to the portability of devices, the desire to make devices smaller has increased. These include portable devices, laptops, mobile phones, tablets, etc. Increasing the efficiency and reducing the size of the

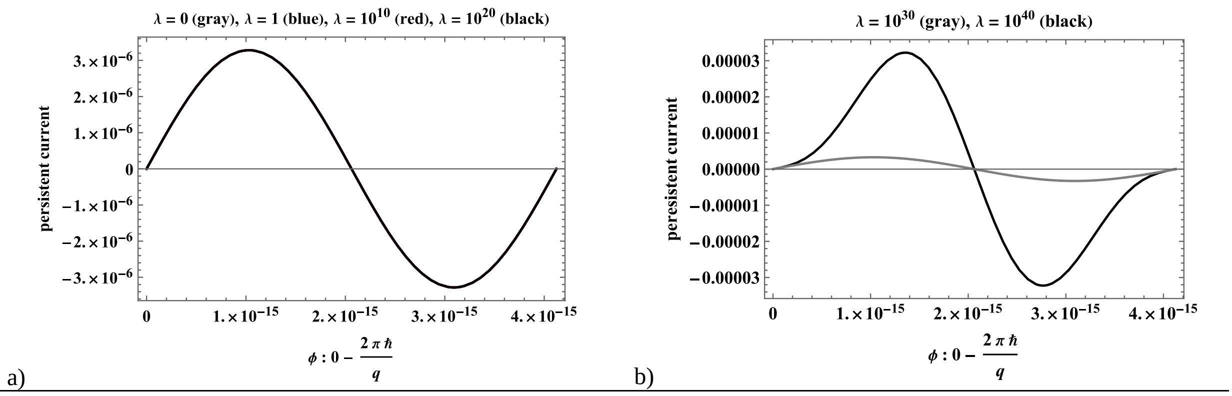

Figure 3. Persistent current of a nonlinear quantum ring vs φ for various λ values.
a) λ=0 - 10^{20} , b) λ=10^{30} and λ=10^{40}
($\hbar = 1.05\times10^{-34} J.s, q_e = 1.6\times10^{-19} C, L = 2\times10^{-10} H$)

converter used in the charger of these devices and the converters used in the devices themselves and increasing the life of the device is of particular importance. Based on, with the progress in nanotechnology and microelectronics, the trend in the miniaturization of integrated electronic devices including inductors and a capacitors towards atomic scale dimensions in the charge discreetness conditions becomes strong and definite. In order to increase the efficiency of the inductors and due to the importance electrical charge discreetness, we have generalized LC mesoscopic quantum electrical circuits that include nonlinear inductors based on Li and Chen theory. By ADM approach, a pure quantum relation for the persistent current in the nonlinear electrical LC-circuits is found. This relation is a periodic function of the magnetic flux with the fundamental parameter ($\hbar/e$). Since the core material of the nonlinear inductor is usually made of ferromagnetic material. Thus, a quantum mesoscopic ring with ferromagnetic core that is a natural pure nonlinear L design is introduced. The current and energy spectrum of a nonlinear pure L design of a nonlinear pure L design are obtained and shown that these are a pure quantum characteristic. Therrfore, with the ADM can obtained the quantum analtycal equations and can said that it is a powerful mathematical tool for solving quantum nonlinear differential equations.

Appendix

Linear and nonlinear differential equations appear in many engineering and science problems. Many of these problems can be solved using analytical and numerical methods, but there are other problems that cannot be solved using these methods, so semi-analytical methods can be a suitable alternative to solve these problems. In this method, the solution of the equation is obtained as an infinite series, which usually converges to the real solution or an approximation of the real solution.

In the ADM method parameter L as second-order derivative of the variable t, is introduced then L^{-1} (now L^{-1} is simply an n-fold integration for a nth order L) is defined as follows (2,8)

$$L^{-1}() = \int_0^t \int_0^t ()dtdt \quad (A1)$$

$$Lp = -\frac{1}{LC}p - \frac{4\lambda}{C}p^3 \quad (A2)$$

$$L^{-1}Lp = -\frac{1}{LC}L^{-1}p - \frac{4\lambda}{C}L^{-1}p^3 \quad (A3)$$

In this expression, p^3 is a nonlinear sentence that is replaced with the $\sum_{n=0}^{\infty} A_n$ series i.e.

$$\begin{aligned} A_0 &= p_0^3 \\ A_1 &= 3p_0^2 p_1 \\ A_2 &= 3p_0^2 p_2 + 3p_1^2 p_0 \\ A_3 &= p_1^3 + 3p_0^2 p_3 + 3p_0 p_1 p_2 \\ &. \\ &. \\ &. \end{aligned} \quad (A4)$$

Where A_n are polynomials of Adomian. In the following, we replace the $\sum_{n=0}^{\infty} p_n$ series instead of p operator, then the equation (A3) is given by

$$\sum_{n=0}^{\infty} p_n = p_0 - \frac{1}{LC}L^{-1}(\sum_{n=0}^{\infty} p_n) - \frac{4\lambda}{C}L^{-1}(\sum_{n=0}^{\infty} p_n)^3 \quad (A5)$$

where

$$\begin{aligned} &p_0 = p(0) \\ &p_{n+1} = -\frac{1}{LC}\int_0^t \int_0^t p_n dtdt - \frac{4\lambda}{C}\int_0^t \int_0^t A_n dtdt, \ n=0, 1, 2, ... \end{aligned} \quad (A6)$$

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